BASIC CIRCUITS ANALYSIS

OHM'S LAW :-

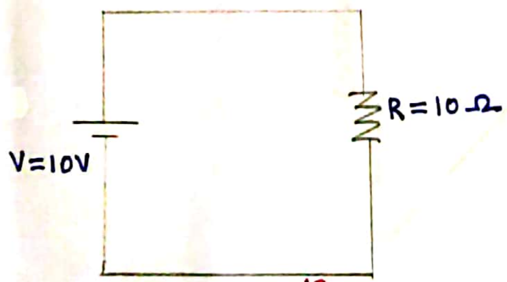
The relationship between Voltage (V), current (I) and resistance (R) was first discovered by scientist George Simon Ohm. Ohm determined experimentally that,

> Current in a resistive circuit is directly proportional to its applied voltage and inversely proportional to its resistance.
>
> $$I = \frac{V}{R}$$

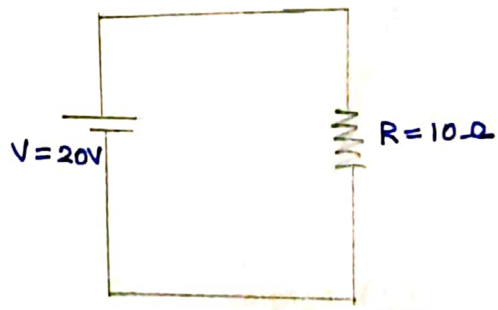
where

$I \rightarrow$ current in ampere
$V \rightarrow$ Voltage in Volts
$R \rightarrow$ Resistance in ohms.

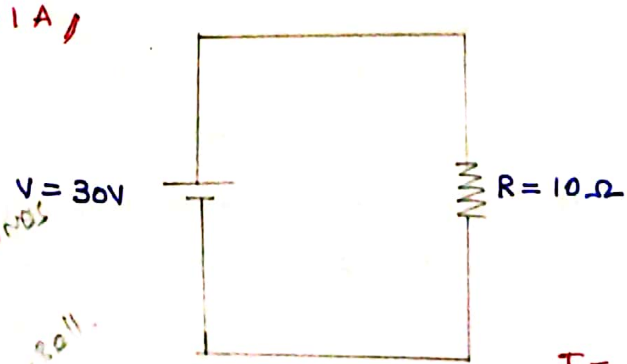
$I = \frac{V}{R}$
$V = IR$
$R = \frac{V}{I}$



$I = \frac{V}{R} \Rightarrow \frac{10}{10}$
$\Rightarrow 1A$



$I = \frac{20}{10} = 2A$



$I = \frac{30}{10}$
$= 3A$

Refer Pg No. 102 Robinns & Pg No. 37 David A Bell

$I \propto V$

POWER

Power is defined as the rate of doing work

$$\text{Power} = \frac{\text{Work Done (or) energy}}{\text{Time}}.$$

POWER IN ELECTRICAL & ELECTRONIC SYSTEMS

Power = Voltage × current

$$\boxed{P = V \times I}$$

The unit of electric power is watt (W)
Motors are rated in horse power (or watts)

Also,

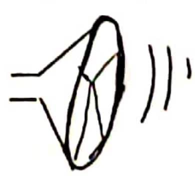
$$\boxed{P = \frac{V^2}{R} \qquad P = I^2 R}$$

- the greater the power rating of the heater, the more heat energy it can produce per second.
- larger the power rating of the motor, the more the mechanical work it can do per second.

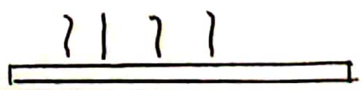
Also,

- Power dissipated in a lamp is converted in to light

- power supplied to a speaker is converted in to sound.



- Power dissipated in conductor is wasted power



ENERGY

Energy is defined as capacity to do work.

Electrical energy = Power × Time.

→ Energy consumption is measured in watt hour (Wh)

For example,

✓ If you run a 100-W lamp for 1 hour, the energy consumed is (100 W) × (1 hour) = 100 Wh

✓ If you run a 1500-W electric heater for 12 hours, the energy consumed is (1500 W) × (12 hours) = 18 000 Wh
= 18 KWh
= 18 unit

$$\boxed{1\,\text{kWh} = 1\,\text{unit}}$$

Problems :-

1) A 27 Ω resistor is connected to a 12-V battery. What is the current?

Soln

$R = 27\,\Omega$

$V = 12$

$I = ?$

$$\boxed{I = \frac{V}{R}}$$

$$I = \frac{12}{27}$$

$$\boxed{I = 0.444\ A}$$

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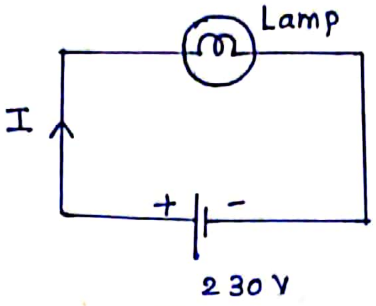
2. Find the resistance of the lamp if the electric lamp draws 4 A at 230V as shown in Figure.



By ohms law,

$$V = IR$$

$$R = \frac{V}{I}$$

$$R = \frac{230}{4}$$

$$\boxed{R = 57.5\,\Omega}$$

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3. Calculate the current supplied to a 100 W lamp with 115 V supply.

<u>Soln</u>

given

$P = 100\,W$

$V = 115\,V$

$I = ?$

$$P = V \times I$$

$$I = \frac{P}{V}$$

$$= \frac{100}{115}$$

$$\boxed{I = 0.86\,A}$$

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4. An electric heater draws 8 A from a 250 V supply. What is its power rating? Also find the resistance of heater element?

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$P = V \times I$

$= 250 \times 8$

$\boxed{P = 2000\,W.}$

$R = \frac{V}{I}$

$= \frac{250}{8}$

$\boxed{R = 31.25\,\Omega}$

5. How much energy does a 100 W electric bulb consume in two hours ?

soln

given,

$P = 100\,W$

$t = 2$ hours.

$E = P \times t$

$= 100 \times 2$

$\boxed{E = 200\,Wh}$

$E = P \times t$

$= 100 \times 2 \times 60 \times 60$

$E = 720000$

6. Determine the total energy used by a 100 W lamp for 12 hours and 1.5 KW heater for 45 minutes.

soln

Convert all quantities to the same set of units

$1.5\,KW \Rightarrow 1500\,W$

45 minutes $\Rightarrow$

$\frac{45}{60} \Rightarrow 0.75$ hour

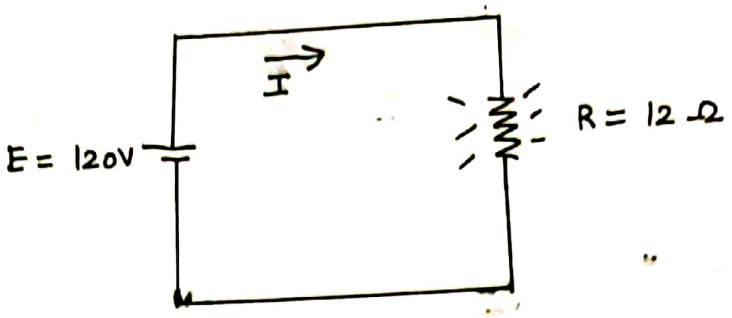
$W = (100 \times 12) + (1500 \times 0.75)$

$= 2325\,Wh$

$= 2.325\,KWh$

$= 2.325$ units.

7. Compute the power supplied to the electric heater of figure using all three electrical power formulas.



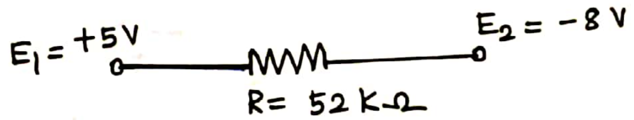
$$I = \frac{V}{R} = \frac{120}{12} = 10\,A$$

$$P = V \times I = 120 \times I = 120 \times 10 = 1200\ W$$

$$P = \frac{V^2}{R} = \frac{120^2}{12} = 1200\ W$$

$$P = I^2 R = 10^2 \times 12 = 1200\ W$$

8. Determine the current and direction in the circuit shown in Fig.

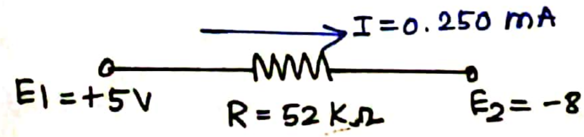


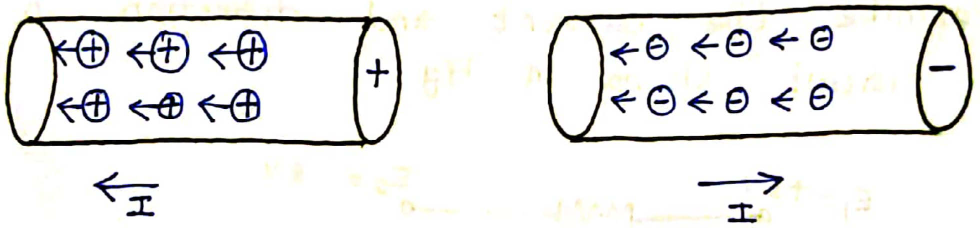
Voltage,

$$= 5 - (-8) = 5 + 8 = 13\ V$$

current,

$$I = \frac{13}{52 \times 1000} = 0.250\ mA$$





Note that : current flow is in the direction of positive charge (+)
&
current flow is in the opposite direction of negative charge.

CURRENT

current is defined as the rate of flow (or rate of movement) of charge.

$$I = \frac{Q}{t}$$

The symbol for current is I

where, $Q \rightarrow$ charge in coulombs.
$t \rightarrow$ time.

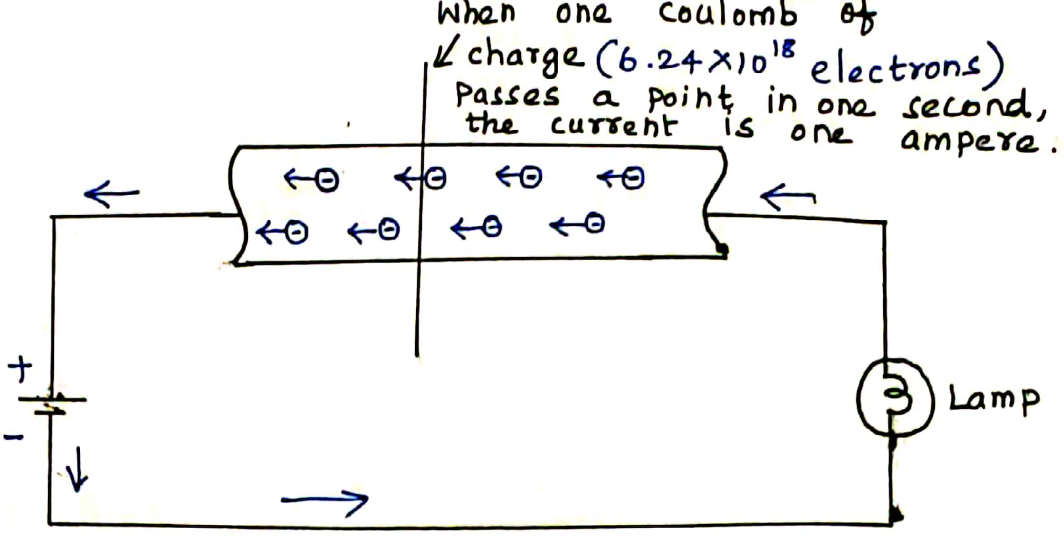
The current may be due to motion of either positive or negative charge or both in a circuit.

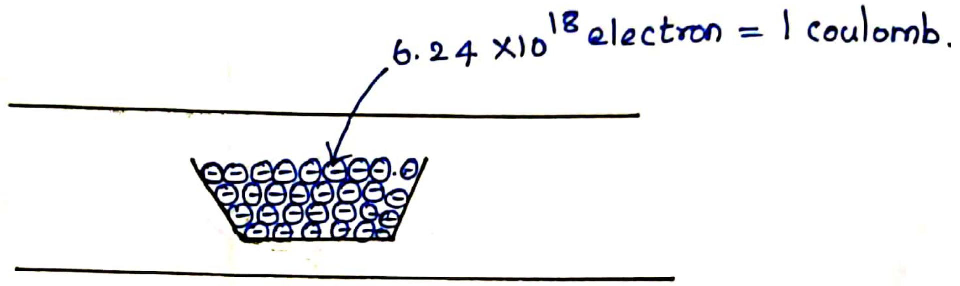
✓ In metal, the electron motion causes current.

→ In metals like Copper, there are large number of free electrons.

→ These electrons move randomly throughout the material, but their net movement in any given direction is zero.

→ Assume now, that a battery is connected. since the electrons are attracted by the positive pole of the battery and repelled by the negative pole, they move around the circuit passing through the wire, the lamp and the battery.
This movement of charge is current.





→ In semiconductor, current is due to both electrons and positively charged holes.

1. How much charge is represented by 4600 electrons?

Charge of a electron = -1.602×10^{-19} c

charge of 4600 electron =

$4600 \times -1.602 \times 10^{-19}$

$= -7.369 \times 10^{-16}$ c //

2. calculate the amount of charge represented by two million protons.

charge of proton = $+1.602 \times 10^{-19}$ c

charge of 2 million proton = $20,000,00 \times 1.602 \times 10^{-19}$

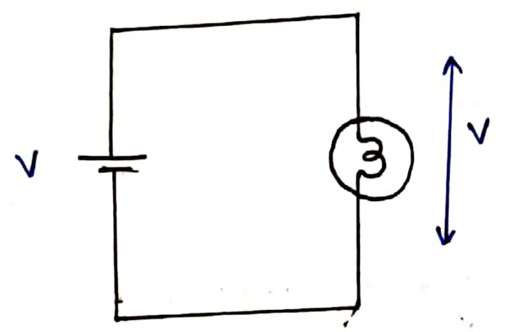
$= 3.204 \times 10^{-13}$ c

VOLTAGE (or) POTENTIAL DIFFERENCE

Voltage is the force that pushes the electron through a given circuit.

→ It is measured in volts.

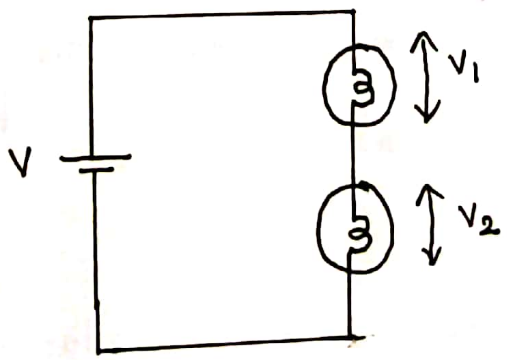
VOLTAGE RISE & VOLTAGE DROP (or) POTENTIAL RISE & POTENTAL DROP



→ Battery generates potential or voltage rise

→ The voltage across the bulb is potential drop.

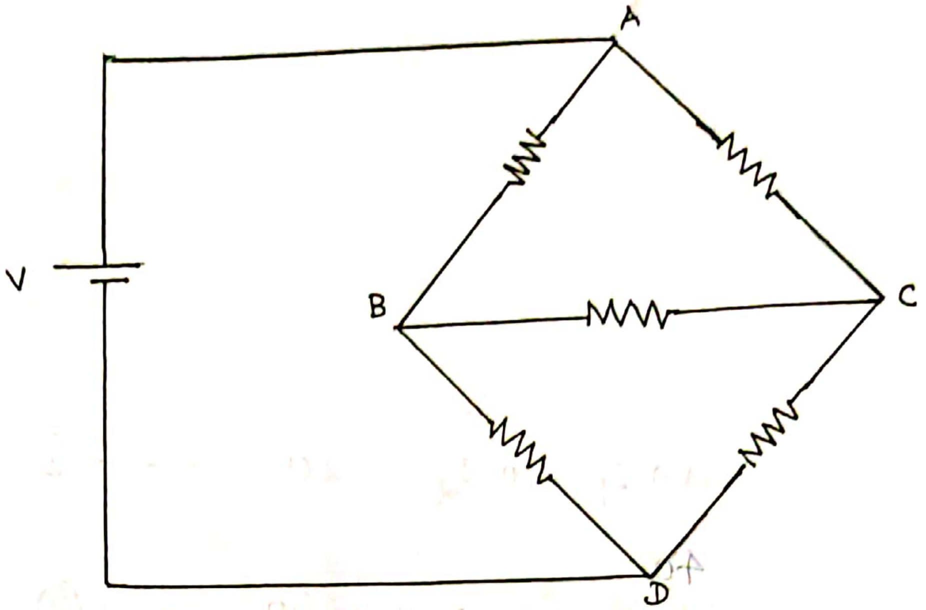
If you put two identical bulbs in series.



$V = V_1 + V_2$

→ Each bulb share the voltage drop.

ELECTRIC CIRCUIT TERMS



ACTIVE ELEMENT :

The sources of energy are called active element.

eg) Battery, Generator.

PASSIVE ELEMENT :

Element that dissipates the energy are called passive element.

eg) Resistor, capacitor and inductor.

NODE

A point where two or more ~~elements~~ branches joined together

A, B, C, D are nodes.

BRANCH

Elements connected between two nodes

AB, BC, CD, DB, ABCDA, : ABC, BCD are loops.

MESH

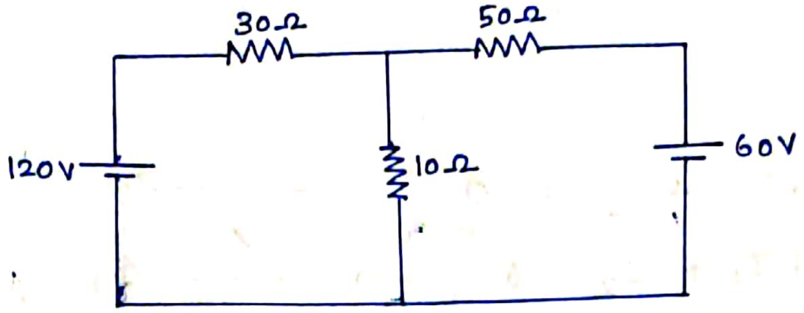
A loop that does not contain any other loop with in it.

LOOP

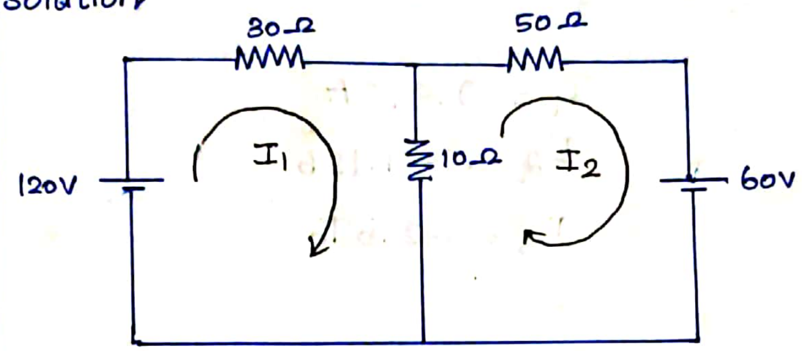
closed path for the flow of current is called as loop.

MESH ANALYSIS [DC CIRCUITS]

1. Solve the mesh currents shown in Fig.



Solution



Mesh 1

$$30I_1 + 10(I_1 - I_2) - 120 = 0$$

$$30I_1 + 10I_1 - 10I_2 = 120$$

$$40I_1 - 10I_2 = 120 \quad \text{——— (1)}$$

Mesh 2

$$50I_2 + 60 + 10(I_2 - I_1) = 0$$

$$50I_2 + 60 + 10I_2 - 10I_1 = 0$$

$$-10I_1 + 60I_2 = -60 \quad \text{——— (2)}$$

$$40I_1 - 10I_2 = 120$$

$$-10I_1 + 60I_2 = -60$$

solving

$$I_1 = 2.86\ A$$

$$I_2 = -0.521$$

Negative sign of I_2 Indicates the direction of I_2 is anticlock wise

$$I_2 = 0.521$$

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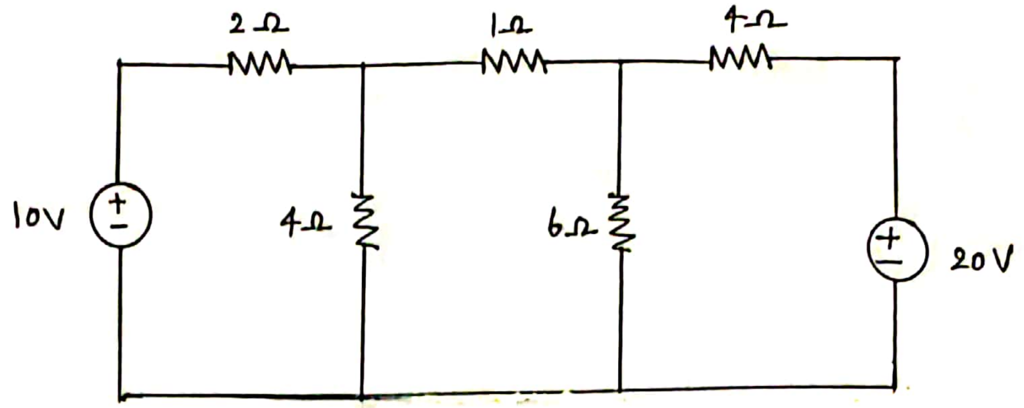
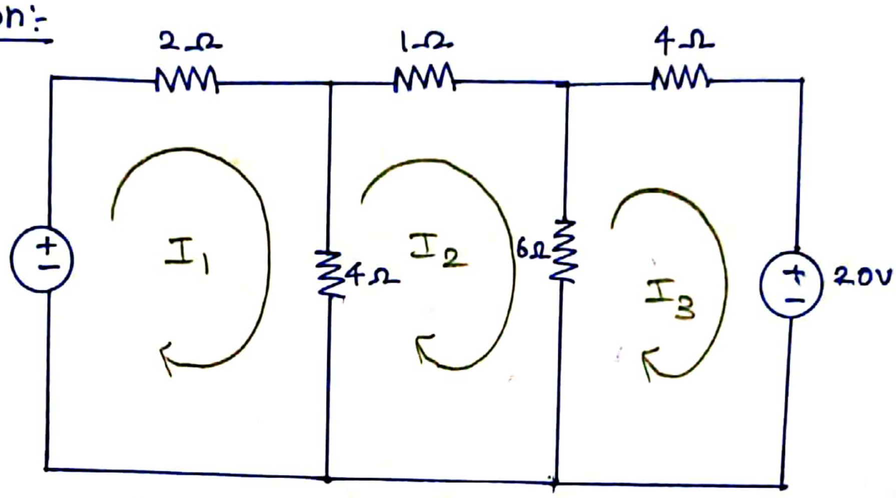
2)



calculate current through 6Ω resistance using loop analysis.

Solution:-



Mesh 1 (KVL)

$2I_1 + 4(I_1 - I_2) - 10 = 0$

$2I_1 + 4I_1 - 4I_2 = 10$

$6I_1 - 4I_2 = 10$ ——— ①

Mesh 2 (KVL)

$1I_2 + 6(I_2 - I_3) + 4(I_2 - I_1) = 0$

$I_2 + 6I_2 - 6I_3 + 4I_2 - 4I_1 = 0$

$-4I_1 + 11I_2 - 6I_3 = 0$ ——— ②

Mesh 3 (KVL)

$4I_3 + 20 + 6(I_3 - I_2) = 0$

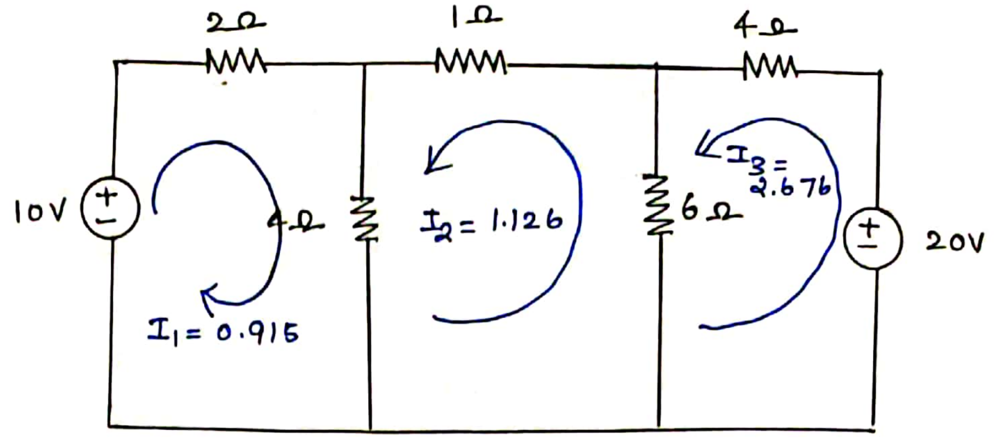
$4I_3 + 20 + 6I_3 - 6I_2 = 0$

$-6I_2 + 10I_3 = -20$ ——— ③

Solving, $I_1 = 0.915\text{ A}$

$I_2 = -1.126\text{ A}$

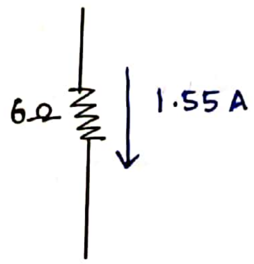
$I_3 = -2.676\text{ A}$



current through 6Ω resistance $= I_3 - I_2$

$= 2.676 - 1.126$

$= 1.55$ A



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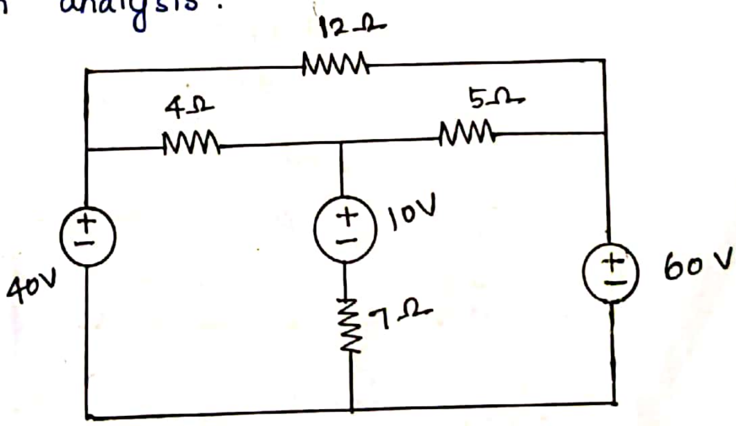
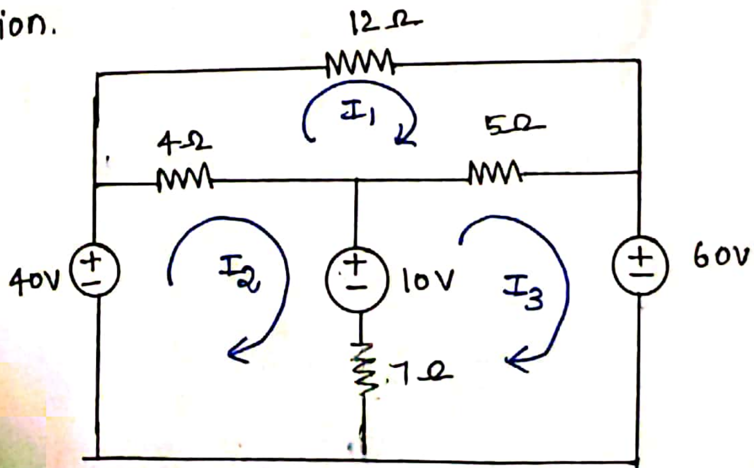
ex 1.42
1.63

3. Solve the current in 12Ω resistor by mesh analysis.



Solution.



Mesh 1 (KVL)

$$12 I_1 + 5(I_1 - I_3) + 4(I_1 - I_2) = 0$$

$$12 I_1 + 5 I_1 - 5 I_3 + 4 I_1 - 4 I_2 = 0$$

$$21 I_1 - 4 I_2 - 5 I_3 = 0 \quad \text{——} \; (1)$$

Mesh 2 (KVL)

$$4(I_2 - I_1) + 10 + 7(I_2 - I_3) - 40 = 0$$

$$4 I_2 - 4 I_1 + 10 + 7 I_2 - 7 I_3 - 40 = 0$$

$$-4 I_1 + 11 I_2 - 7 I_3 = 30 \quad \text{——} \; (2)$$

Mesh 3 (KVL)

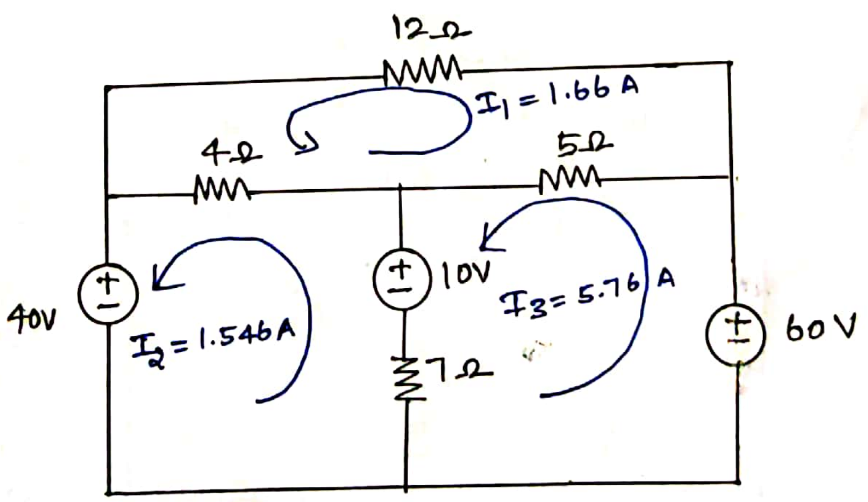
$$5(I_3 - I_1) + 60 + 7(I_3 - I_2) - 10 = 0$$

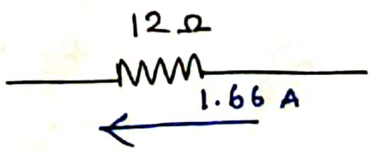
$$5 I_3 - 5 I_1 + 60 + 7 I_3 - 7 I_2 - 10 = 0$$

$$-5 I_1 - 7 I_2 + 12 I_3 = -50 \quad \text{——} \; (3)$$

solving (1), (2) & (3) we get

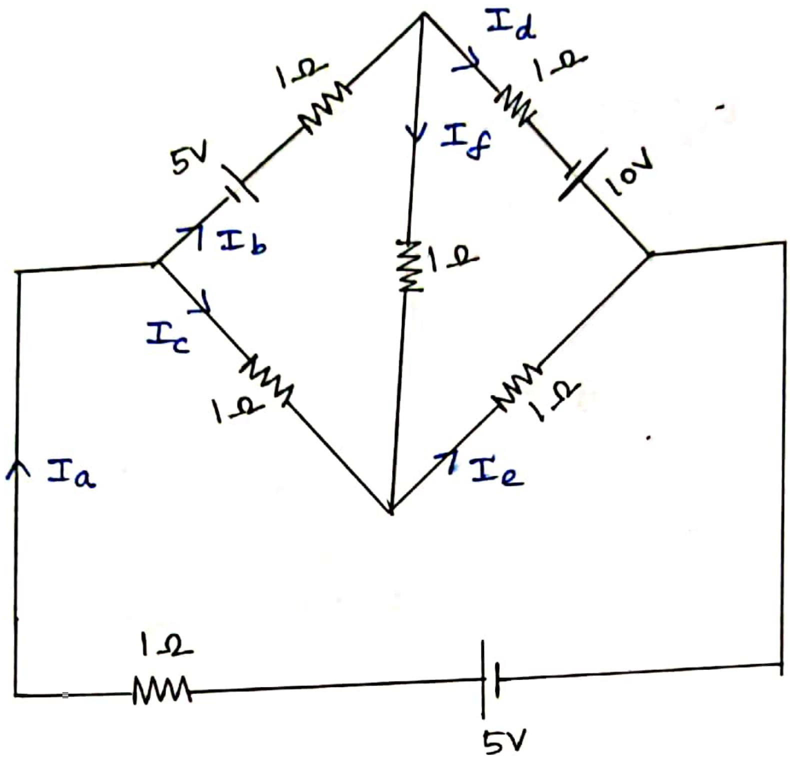
$I_1 = -1.66$ A
$I_2 = -1.546$ A
$I_3 = -5.76$ A



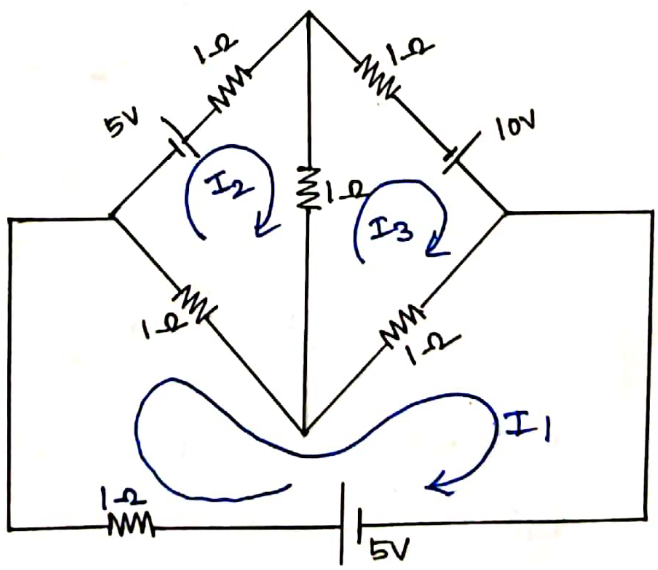


$$\boxed{I_{12\Omega} = 1.66 \text{ A}}$$

4) Determine the currents in various elements ($I_a, I_b, I_c, I_d, I_e, I_f$) of the bridge circuit shown in Fig. by mesh Analysis.



Solution:-



Mesh 1 (KVL)

$$1 I_1 + 1(I_1 - I_2) + 1(I_1 - I_3) - 5 = 0$$

$$I_1 + I_1 - I_2 + I_1 - I_3 - 5 = 0$$

$$3I_1 - I_2 - I_3 = 5 \quad \text{——— (1)}$$

Mesh 2 (KVL)

$$1I_2 + 1(I_2 - I_3) + 1(I_2 - I_1) - 5 = 0$$

$$I_2 + I_2 - I_3 + I_2 - I_1 = 5$$

$$-I_1 + 3I_2 - I_3 = 5 \quad \text{——} \quad (2)$$

Mesh 3 (KVL)

$$1(I_3 - I_2) + 1I_3 - 10 + 1(I_3 - I_1) = 0$$

$$I_3 - I_2 + I_3 - 10 + I_3 - I_1 = 0$$

$$-I_1 - I_2 + 3I_3 = 10 \quad \text{——} \quad (3)$$

solving (1), (2) & (3) we get

$$I_1 = 6.25 \text{ A}$$
$$I_2 = 6.25 \text{ A}$$
$$I_3 = 7.5 \text{ A}$$

$$I_a = I_1$$

$$\Rightarrow \boxed{I_a = 6.25\text{ A}}$$

$$I_b = I_2 \Rightarrow \boxed{I_b = 6.25\text{ A}}$$

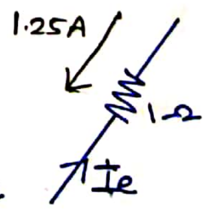
$$I_c = I_1 \sim I_2$$

$$\Rightarrow \boxed{I_c = 0}$$

$$I_d = I_3$$

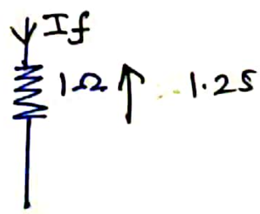
$$\boxed{I_d = 7.5\text{ A}}$$

$$I_e = I_3 \sim I_1$$
$$= 7.5 - 6.25$$
$$I_e = 1.25 \text{ A}$$



$$\boxed{I_e = -1.25\text{A}}$$

$$I_f = I_2 \sim I_3$$
$$= I_3 - I_2$$
$$= 7.5 - 6.25$$
$$= 1.25$$



$$\Rightarrow \boxed{I_f = -1.25\text{A}}$$

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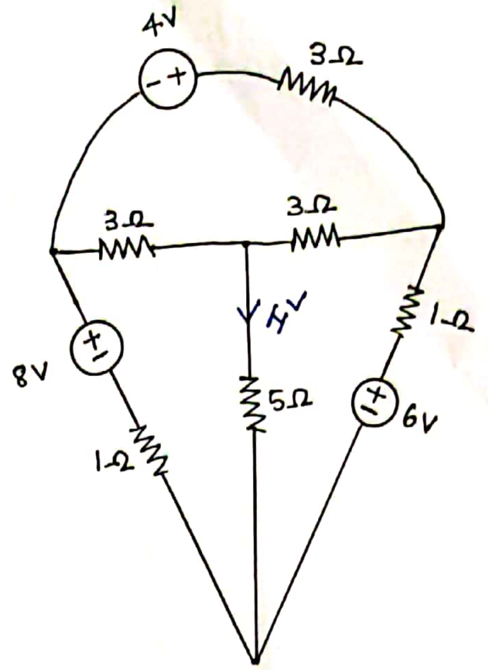
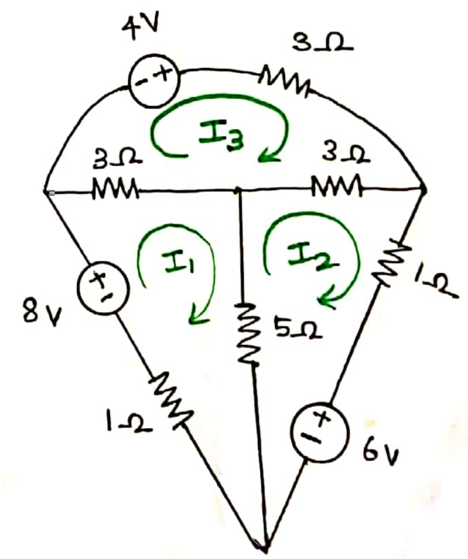
5) Determine current I_L



Solution:-



Mesh 1

$$3(I_1 - I_3) + 5(I_1 - I_2) + 1 I_1 - 8 = 0$$

$$3I_1 - 3I_3 + 5I_1 - 5I_2 + I_1 = 8$$

$$9I_1 - 5I_2 - 3I_3 = 8 \quad \text{———} \quad (1)$$

Mesh 2

$$3(I_2 - I_3) + 1 I_2 + 6 + 5(I_2 - I_1) = 0$$

$$3I_2 - 3I_3 + I_2 + 6 + 5I_2 - 5I_1 = 0$$

$$-5I_1 + 9I_2 - 3I_3 = -6 \quad \text{———} \quad (2)$$

Mesh 3

$$3I_3 + 3(I_3 - I_2) + 3(I_3 - I_1) = 0$$

$$3I_3 + 3I_3 - 3I_2 + 3I_3 - 3I_1 \quad -4 = 0$$

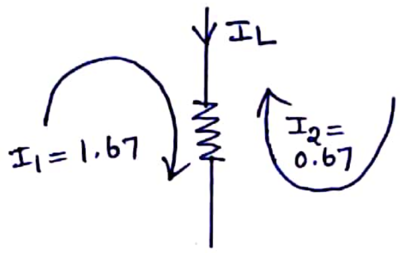
$$-3I_1 - 3I_2 + 9I_3 = 4 \quad \text{——} \quad (3)$$

Solving (1), (2) & (3)

$I_1 = 1.67$ A

$I_2 = 0.67$ A

$I_3 = 1.22$ A

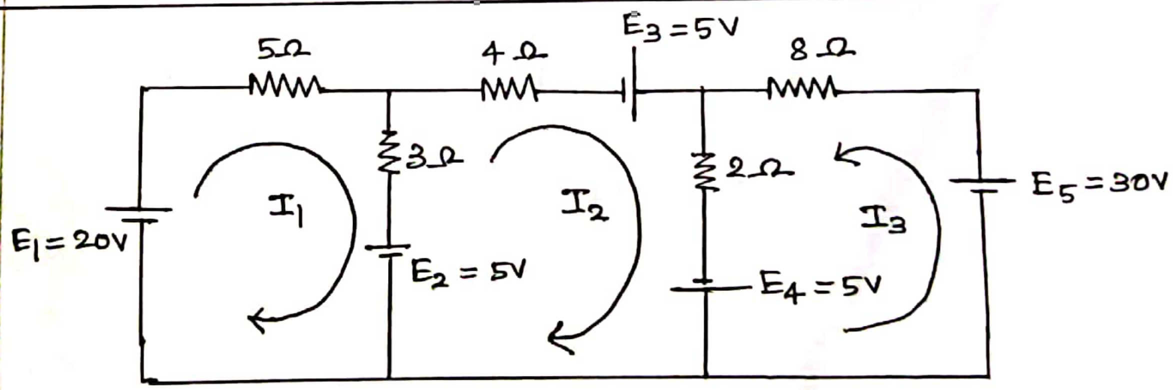


$$I_L = I_1 - I_2$$

$$= 1.67 - 0.67$$

$$\boxed{I_L = 1 \text{ A}}$$

6.



Find the current supplied by each battery.

Mesh 1

$$-20 + 5I_1 + 3(I_1 - I_2) + 5 = 0$$

$$-20 + 5I_1 + 3I_1 - 3I_2 + 5 = 0$$

$$8I_1 - 3I_2 = 15 \quad \text{——} \quad (1)$$

Mesh 2

$$-5+2(I_2+I_3)-5-5+3(I_2-I_1)+4I_2=0$$

$$-5+2I_2+2I_3-10+3I_2-3I_1+4I_2=0$$

$$-3I_1+9I_2+2I_3=15 \quad \text{—— ②}$$

Mesh 3

$$-30+8I_3+2(I_3+I_2)-5=0$$

$$-30+8I_3+2I_3+2I_2-5=0$$

$$2I_2+10I_3=35 \quad \text{—— ③}$$

solving ①, ② & ③ we get

$I_1 = 2.55\text{ A}$

$I_2 = 1.82\text{ A}$

$I_3 = 3.13\text{ A}$

current supplied by Battery

$E_1 = I_1 = 2.55\text{ A}$ //

$E_2 = I_1 - I_2 = 2.55 - 1.82 = 0.73\text{ A}$ //

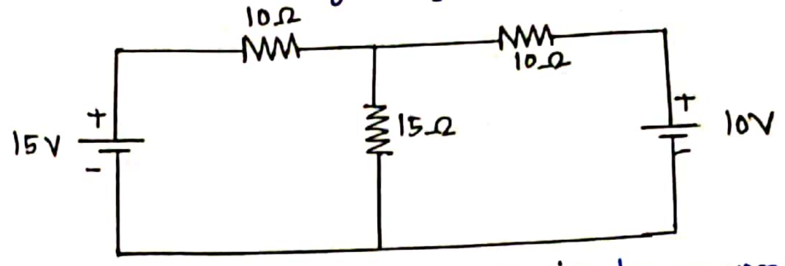
$E_3 = I_2 = 1.82\text{ A}$ //

$E_4 = I_2 + I_3 = 4.95\text{ A}$ //

$E_5 = I_3 = 3.13\text{ A}$ //

NODAL ANALYSIS [DC]

1. Find the current through 15Ω resistor in the network shown in Fig by nodal method.

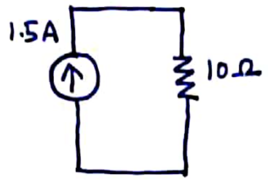


<u>soln</u> convert the voltage source in to current source.

$I = \frac{V}{R}$

$= \frac{15}{10}$

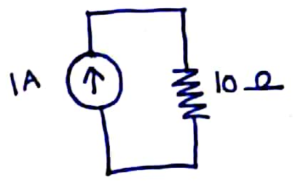
$= 1.5A$

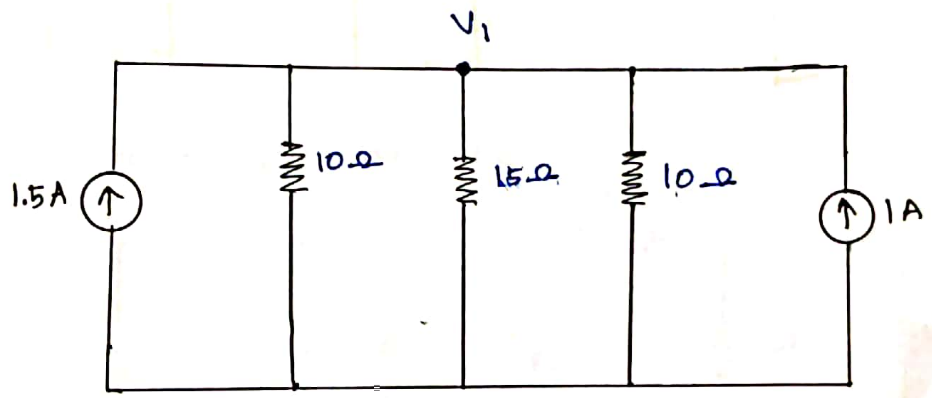


$I = \frac{V}{R}$

$= \frac{10}{10}$

$= 1A$





<u>Node 1 (KCL)</u>

$$1.5 + 1 = \frac{V_1}{10} + \frac{V_1}{15} + \frac{V_1}{10}$$

$$2.5 = V_1\left(\frac{1}{10} + \frac{1}{15} + \frac{1}{10}\right)$$

$$0.267\ V_1 = 2.5$$

$$V_1 = \frac{2.5}{0.267}$$

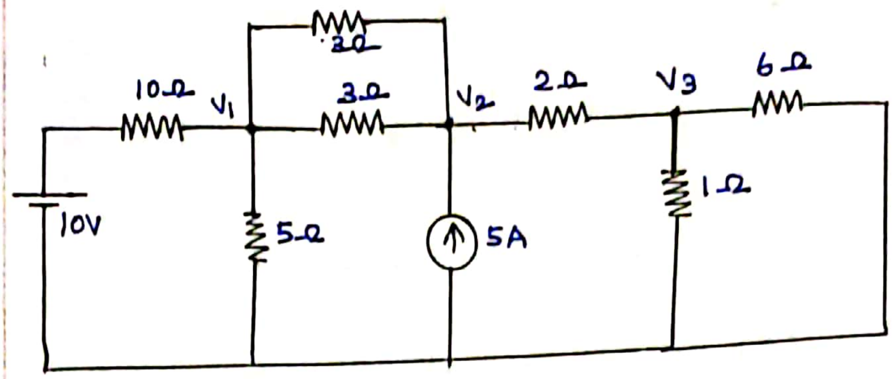
$$V_1 = 9.36V$$

current through 15Ω resistor is

$$I_{15} = \frac{9.36}{15} \qquad \left[I = \frac{V}{R}\right]$$

$$\boxed{I_{15} = 0.624\ A}$$

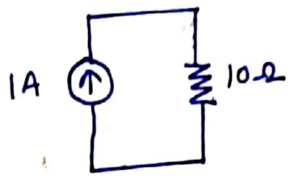
2. Determine the voltage at each node for the given circuit.

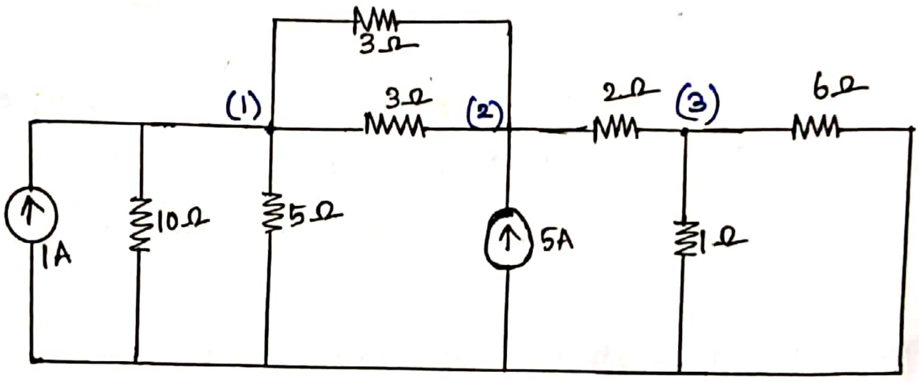


Solution :-

Convert the voltage source in to current source.

$$I = \frac{V}{R} = \frac{10}{10} = 1\,A$$





Node 1 (KCL)

$$1 = \frac{V_1}{10} + \frac{V_1}{5} + \frac{V_1 - V_2}{3} + \frac{V_1 - V_2}{3}$$

$$1 = \frac{V_1}{10} + \frac{V_1}{5} + \frac{V_1}{3} - \frac{V_2}{3} + \frac{V_1}{3} - \frac{V_2}{3}$$

$$V_1\left(\frac{1}{10} + \frac{1}{5} + \frac{1}{3} + \frac{1}{3}\right) - V_2\left(\frac{1}{3} + \frac{1}{3}\right) = 1$$

$$0.966\,V_1 - 0.66\,V_2 = 1 \quad \text{——— (1)}$$

Node 2 (KCL)

$$5 = \frac{V_2 - V_1}{3} + \frac{V_2 - V_1}{3} + \frac{V_2 - V_3}{2}$$

$$5 = \frac{V_2}{3} - \frac{V_1}{3} + \frac{V_2}{3} - \frac{V_1}{3} + \frac{V_2}{2} - \frac{V_3}{2}$$

$$V_1\left(-\frac{1}{3} - \frac{1}{3}\right) + V_2\left(\frac{1}{3} + \frac{1}{3} + \frac{1}{2}\right) - \frac{V_3}{2} = 5$$

$$-0.66\,V_1 + 1.166\,V_2 - 0.5\,V_3 = 5 \quad \text{——— (2)}$$

Node 3 (KCL)

$$\frac{V_3 - V_2}{2} + \frac{V_3}{1} + \frac{V_3}{6} = 0$$

$$\frac{V_3}{2} - \frac{V_2}{2} + \frac{V_3}{1} + \frac{V_3}{6} = 0$$

$$-\frac{V_2}{2} + V_3\left(\frac{1}{2} + 1 + \frac{1}{6}\right) = 0$$

$$-0.5V_2 + 1.66\,V_3 = 0 \quad \text{——— (3)}$$

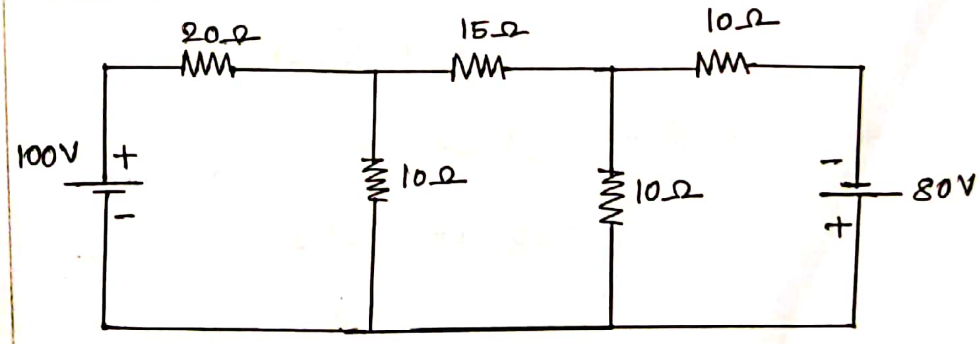
solving (1), (2) & (3) we get

$$\boxed{\begin{aligned} V_1 &= 7.91\ V \\ V_2 &= 10.06\ V \\ V_3 &= 3.03\ V \end{aligned}}$$

3. Calculate the voltage across the 15 Ω resistor in the network shown in Fig. using nodal analysis.

Soln

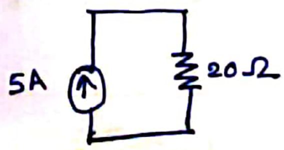
Convert the voltage source in to current source.



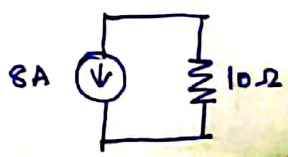
Solution.

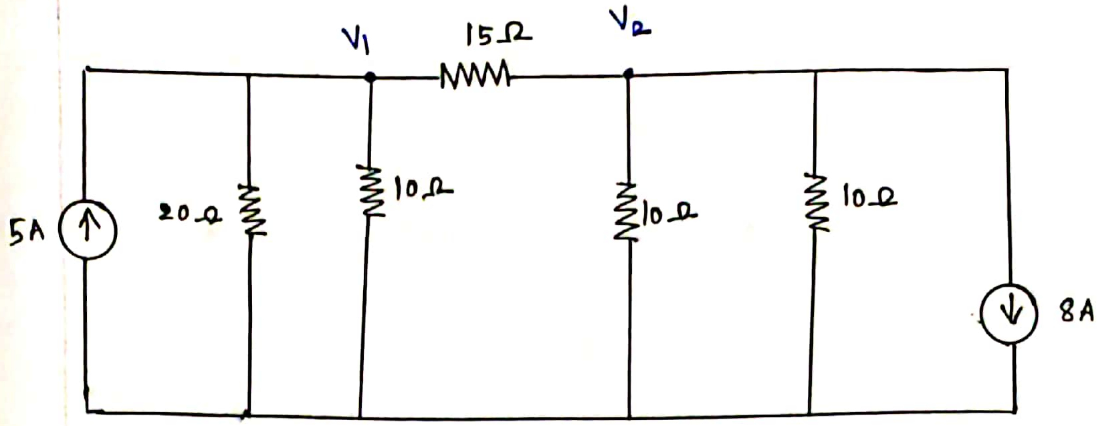
Convert all the voltage sources into current sources.

$$I = \frac{V}{R} = \frac{100}{20} = 5A$$



$$I = \frac{V}{R} = \frac{80}{10} = 8\ A$$





Node 1 (KCL)

$$5 = \frac{V_1}{20} + \frac{V_1}{10} + \frac{V_1 - V_2}{15}$$

$$\frac{V_1}{20} + \frac{V_1}{10} + \frac{V_1}{15} - \frac{V_2}{15} = 5$$

$$V_1\left(\frac{1}{20} + \frac{1}{10} + \frac{1}{15}\right) - \frac{V_2}{15} = 5$$

$$0.216 V_1 - 0.066 V_2 = 5 \quad \text{——— (1)}$$

Node 2 (KCL)

$$\frac{V_2 - V_1}{15} + \frac{V_2}{10} + \frac{V_2}{10} + 8 = 0$$

$$\frac{V_2}{15} - \frac{V_1}{15} + \frac{V_2}{10} + \frac{V_2}{10} + 8 = 0$$

$$-\frac{V_1}{15} + V_2\left(\frac{1}{15} + \frac{1}{10} + \frac{1}{10}\right) = -8$$

$$-0.066 V_1 + 0.266 V_2 = -8 \quad \text{——— (2)}$$

solving (1) & (2) we get.

$V_1 = 15.1$

$V_2 = -26.3V$

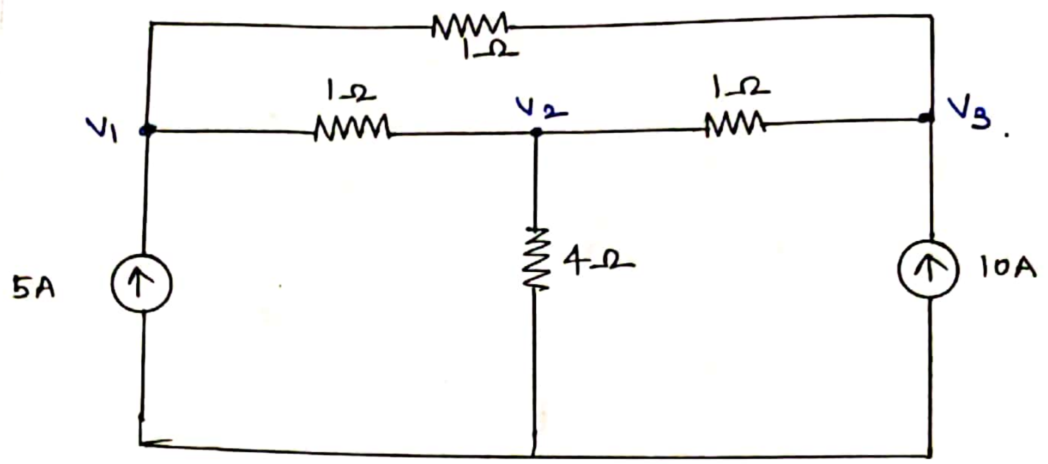
current through 15Ω resistor

$$I_{15} = \frac{V_1 - V_2}{15} \qquad \left[I = \frac{V}{R}\right]$$

$$= \frac{15 - (-26.3)}{15}$$

$$\boxed{I_{15} = 2.75\text{ A}}$$

4. Find V_1, V_2, V_3 by the nodal method for the given circuit.



Node 1 (KCL)

$$5 = \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{1}$$

$$5 = V_1 - V_2 + V_1 - V_3$$

$$2V_1 - V_2 - V_3 = 5 \quad \text{———} \quad (1)$$

Node 2 (KCL)

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{1} + \frac{V_2}{4} = 0$$

$$V_2 - V_1 + V_2 - V_3 + \frac{V_2}{4} = 0$$

$$-V_1 + 2.25V_2 - V_3 = 0 \quad \text{———} \quad (2)$$

Node 3 (KCL)

$$10 = \frac{V_3 - V_2}{1} + \frac{V_3 - V_1}{1}$$

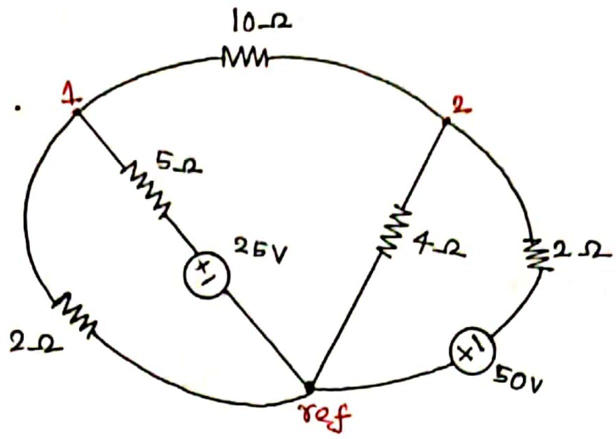
$$10 = V_3 - V_2 + V_3 - V_1 = 0$$

$$-V_1 - V_2 + 2V_3 = 10 \quad \text{———} \quad (3)$$

solving (1), (2) & (3) we get.

$$\boxed{\begin{aligned} V_1 &= 66.6\ V \\ V_2 &= 60\ V \\ V_3 &= 68.33\ V \end{aligned}}$$

5. Solve the network given below by the node voltage method.



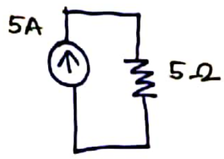
Solution

Convert voltage source in to current source.

$$I = \frac{V}{R}$$

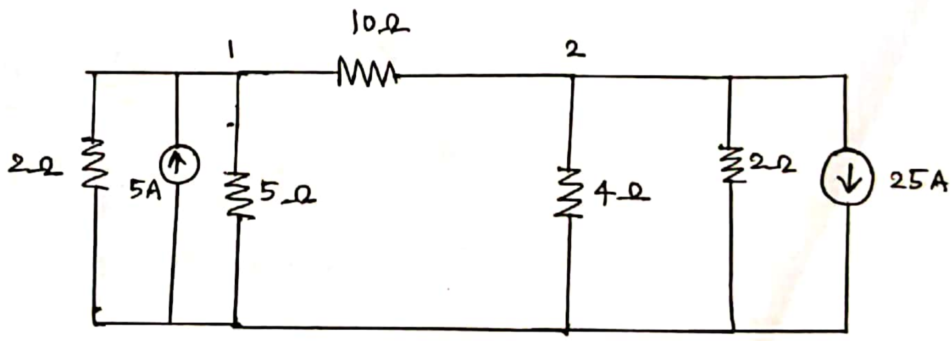
$$= \frac{25}{5}$$

$$= 5A$$



$$I = \frac{V}{R}$$

$$= \frac{50}{2} = 25A$$



Node 1 (KCL)

$$5 = \frac{V_1}{2} + \frac{V_1}{5} + \frac{V_1 - V_2}{10}$$

$$5 = \frac{V_1}{2} + \frac{V_1}{5} + \frac{V_1}{10} - \frac{V_2}{10}$$

$$V_1\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right) - \frac{V_2}{10} = 5$$

$$0.8V_1 - 0.1V_2 = 5 \quad \text{——— (1)}$$

Node 2 (KCL)

$$\frac{V_2 - V_1}{10} + \frac{V_2}{2} + \frac{V_2}{4} + 25 = 0$$

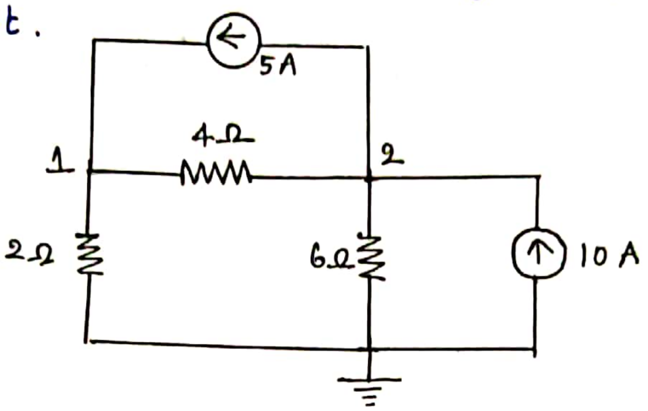
$$\frac{V_2}{10} - \frac{V_1}{10} + \frac{V_2}{2} + \frac{V_2}{4} + 25 = 0$$

$$-\frac{V_1}{10} + V_2\left(\frac{1}{10} + \frac{1}{2} + \frac{1}{4}\right) = -25$$

$$-0.1V_1 + 0.85V_2 = -25$$

$$V_1 = 2.61$$
$$V_2 = -29.10$$

6. Calculate the node voltages of the given circuit.



Soln

Node 1

$$5 = \frac{V_1 - V_2}{4} + \frac{V_1}{2}$$

$$\frac{V_1}{4} - \frac{V_2}{4} + \frac{V_1}{2} = 5$$

$$V_1\left(\frac{1}{4} + \frac{1}{2}\right) - \frac{V_2}{4} = 5$$

$$0.75V_1 - 0.25V_2 = 5 \quad \text{—— (1)}$$

Node 2

$$10 = \frac{V_2 - V_1}{4} + \frac{V_2}{6} + 5$$

$$\frac{V_2}{4} - \frac{V_1}{4} + \frac{V_2}{6} = 10 - 5$$

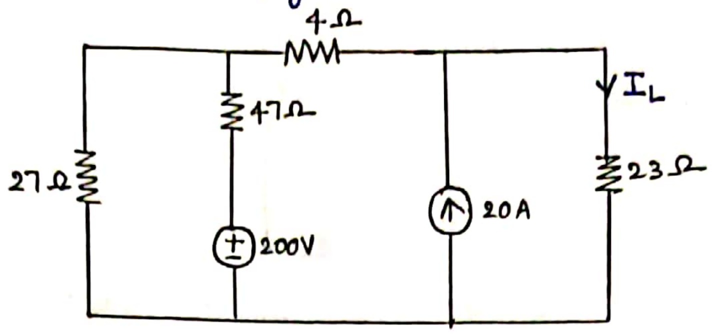
$$-\frac{V_1}{4} + V_2\left(\frac{1}{4} + \frac{1}{6}\right) = 5$$

$$-0.25V_1 + 0.416V_2 = 5 \quad \text{—— (2)}$$

Solving (1) & (2)

$$V_1 = 13.34 \text{ V}$$
$$V_2 = 20 \text{ V}$$

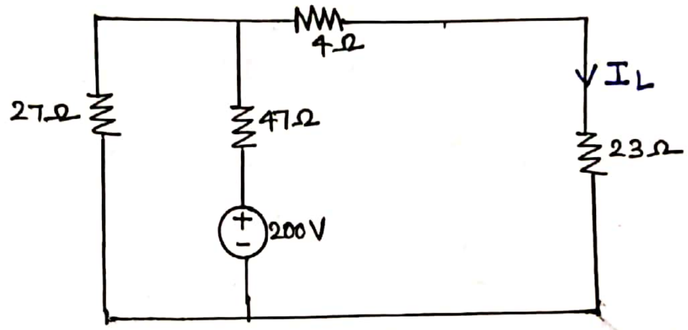
1. Using Superposition theorem, find the current I_L and the power consumed by $23\,\Omega$ resistor in the circuit shown in Fig.



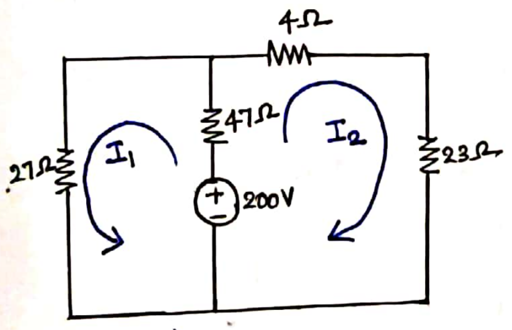
Soln

consider 200V source alone

→ open circuit the current source.



Using Mesh Analysis



Mesh 1 (KVL)

$$-200 + 47(I_1 + I_2) + 27I_1 = 0$$

$$47I_1 + 47I_2 + 27I_1 = 200$$

$$74I_1 + 47I_2 = 200 \quad \text{--- (1)}$$

Mesh 2 (KVL)

$$4I_2 + 23I_2 - 200 + 47(I_2 + I_1) = 0$$

$$4I_2 + 23I_2 + 47I_2 + 47I_1 = 200$$

$$47I_1 + 74I_2 = 200 \quad \text{--- (2)}$$

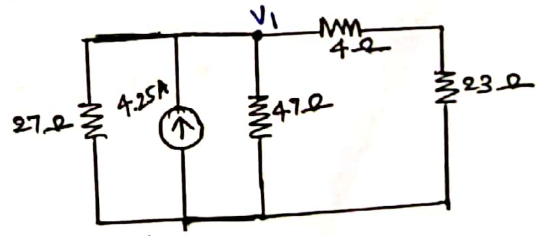
$I_1 = 1.65\,A$

$I_2 = 1.65\,A$

$I_{L_1} = I_2 = 1.65\,A$ /

Using Nodal Analysis.

Convert voltage source in to current source. $I = \frac{V}{R} = \frac{200}{47} = 4.25\,A$



Node 1 (KCL)

$$4.25 = \frac{V_1}{27} + \frac{V_1}{47} + \frac{V_1}{(4+23)}$$

$$4.25 = V_1\left(\frac{1}{27} + \frac{1}{47} + \frac{1}{27}\right).$$

$$0.095\,V_1 = 4.25$$

$$V_1 = \frac{4.25}{0.095}$$

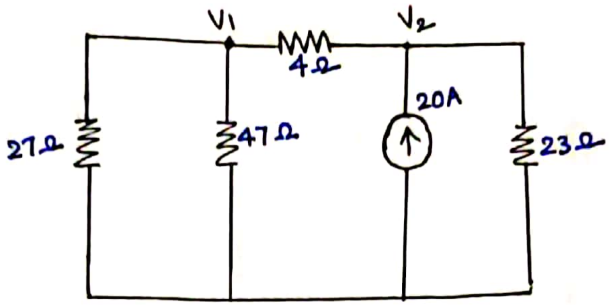
$$V_1 = 44.57\,V$$

$$I_{L_1} = \frac{V_1}{4+23}$$

$$= \frac{44.57}{27} = 1.65\,A \;//$$

consider 20 A source acting alone

short circuit the voltage source.



using Nodal Analysis

Node 1 (KCL)

$$\frac{V_1-V_2}{4}+\frac{V_1}{47}+\frac{V_1}{27}=0$$

$$\frac{V_1}{4}-\frac{V_2}{4}+\frac{V_1}{47}+\frac{V_1}{27}=0$$

$$V_1\left(\frac{1}{4}+\frac{1}{47}+\frac{1}{27}\right)-\frac{V_2}{4}=0$$

$$0.308V_1-0.25V_2=0 \quad \text{— ①}$$

Node 2 (KCL)

$$20=\frac{V_2-V_1}{4}+\frac{V_2}{23}$$

$$\frac{V_2}{4}-\frac{V_1}{4}+\frac{V_2}{23}=20$$

$$-0.25V_1+\left(\frac{1}{4}+\frac{1}{23}\right)V_2=20$$

$$-0.25V_1+0.29V_2=20 \quad \text{— ②}$$

$V_1 = 186.4V$

$V_2 = 229.6V$

$$I_{L2}=\frac{V_2}{23}$$

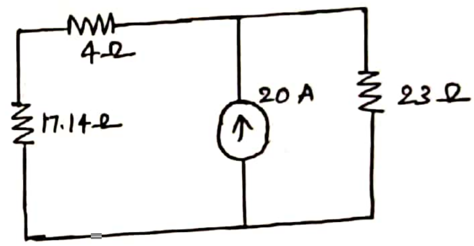
$$=\frac{229.6}{23}$$

$$=9.6A \;//$$

using current division rule

27Ω and 47Ω are in parallel

$$\Rightarrow \frac{27\times 47}{27+47}=17.14\,\Omega.$$



$$I_{L2}=\frac{20\times(4+17.14)}{(4+17.14)+23}$$

$$=\frac{20\times 21.14}{21.14+23}$$

$$=9.578\text{ A} \;//$$

Using Superposition theorem,

$$I_L = I_{L1}+I_{L2}$$

$$=1.65+9.578$$

$$\boxed{I_L = 11.228\text{ A}} \;//$$

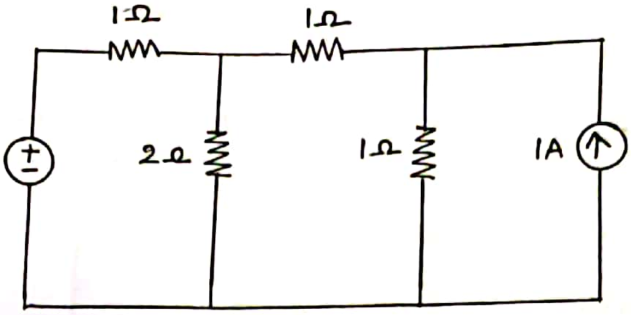
Power consumed by 23 Ω resistor

$$P_{23}=I_L^2 R$$

$$=11.22^2\times 23$$

$$=2895.4\text{ W}$$

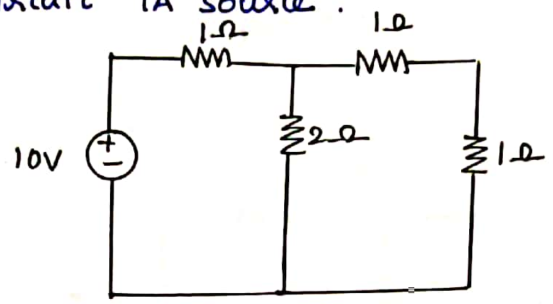
2) Calculate the current through the 2Ω resistor in the circuit shown below, using superposition theorem.



Soln

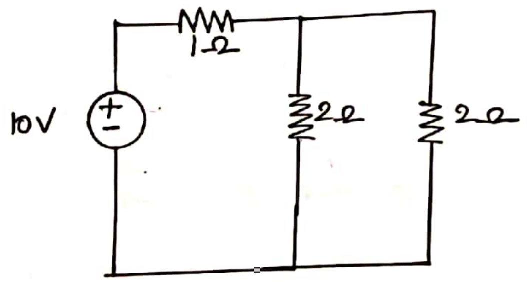
Consider 10V source alone

open circuit 1A source.



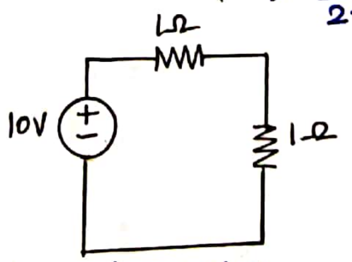
1Ω & 1Ω are in series

$\Rightarrow 1+1 = 2\Omega$



2Ω and 2Ω are parallel

$$\Rightarrow \frac{2\times2}{2+2} = 1\Omega$$



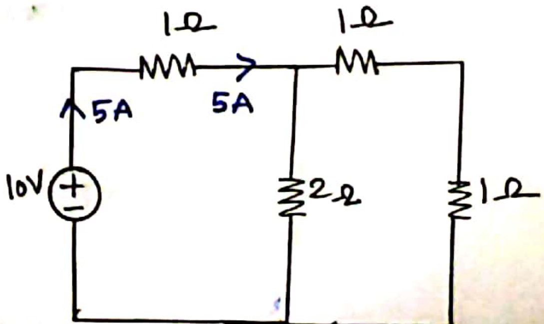
1Ω & 1Ω are in series

$R_{eq} = 2\Omega$

Now $I = \frac{V}{R}$

$= \frac{10}{2}$

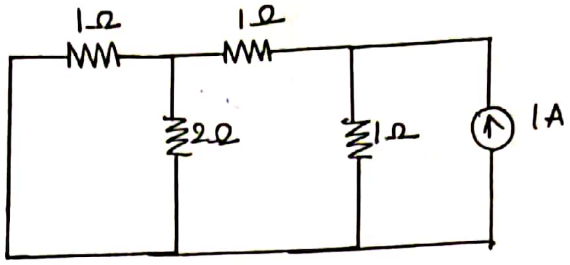
$= 5\,A$



$$I_{2\Omega} = \frac{5 \times 2}{2+2}$$

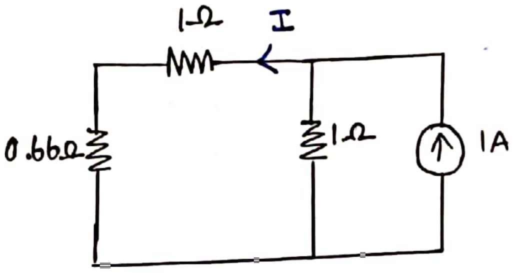
$$I_{2\Omega} = 2.5\,A$$

consider 1A source alone.
short circuit 10V source.

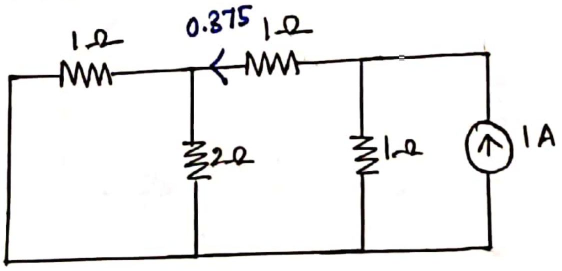


1Ω & 2Ω are in parallel

$$\Rightarrow \frac{1 \times 2}{1+2} = 0.66\,\Omega$$



$$I = \frac{1 \times 1}{1+1.66} = 0.375\,A$$



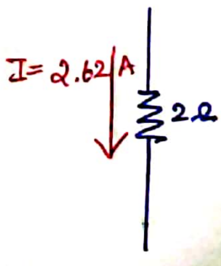
$$I_{2\Omega} = \frac{0.375 \times 1}{1+2}$$

$$I_{2\Omega} = 0.125\,A$$

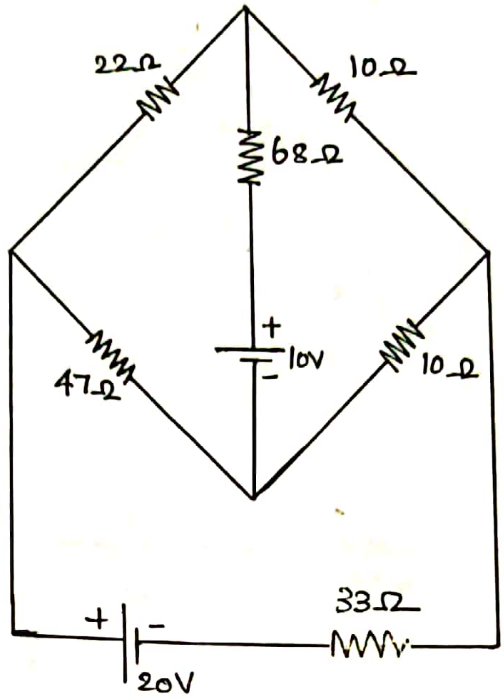
using superposition theorem.

$$I_{2\Omega} = 2.5 + 0.125$$

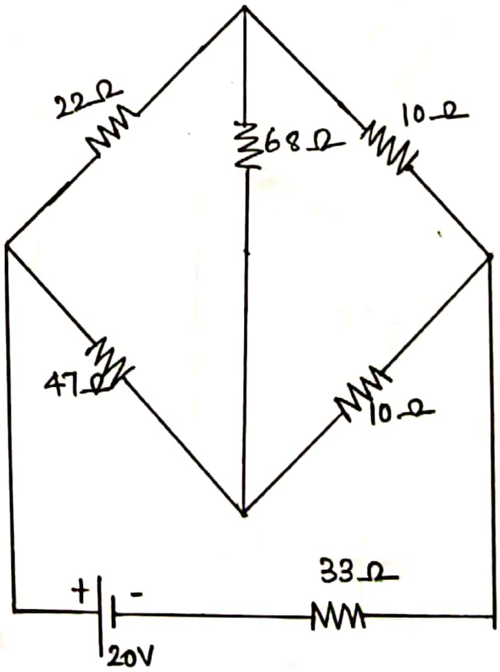
$$\boxed{I_{2\Omega} = 2.625\,A}$$

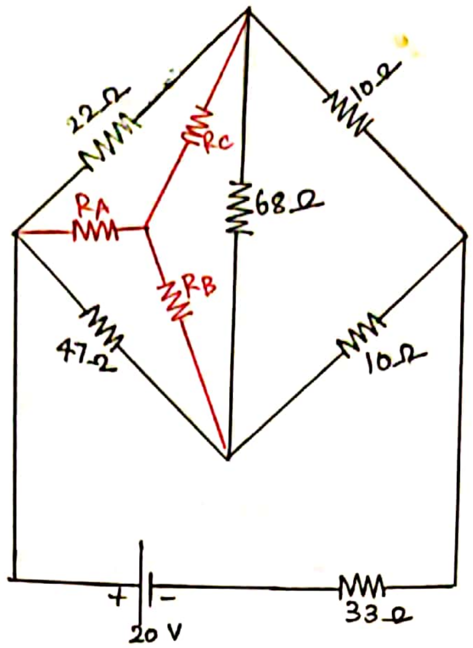


3) Find the power delivered by the 20 V source using superposition theorem.



consider 20 V source alone
short circuit 10V source.

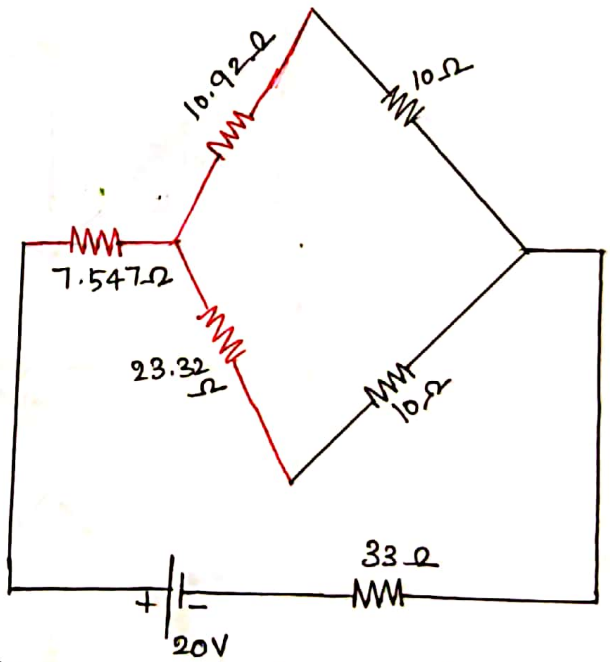




$$R_A = \frac{47 \times 22}{47+22+68}$$
$$= 7.547\ \Omega$$

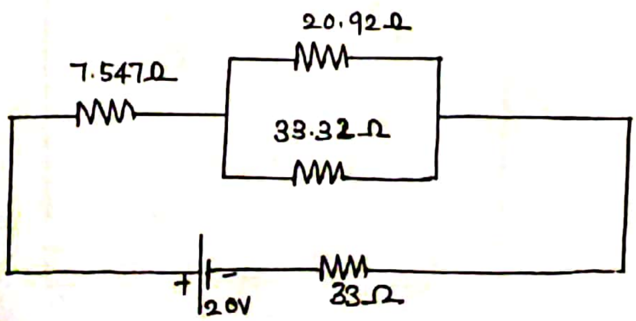
$$R_B = \frac{47 \times 68}{47+22+68}$$
$$= 23.32\ \Omega$$

$$R_C = \frac{22 \times 68}{47+22+68}$$
$$= 10.92\ \Omega$$



10.92 & 10Ω are in series ⇒ $10.92 + 10 = 20.92\ \Omega$

23.32Ω & 10Ω are in series ⇒ $23.32 + 10 = 33.32\ \Omega$



$$R_{eq} = 7.547 + \frac{20.92 \times 33.32}{20.92 + 33.32} + 33$$

$$R_{eq} = 7.547 + 12.85 + 33$$

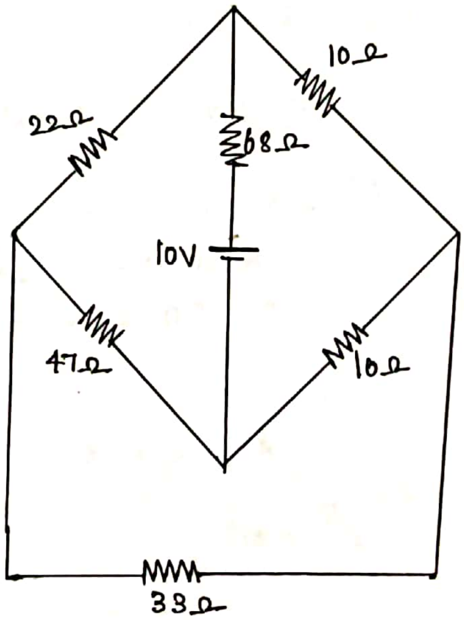
$$R_{eq} = 53.39\Omega.$$

$$I = \frac{V}{R}$$

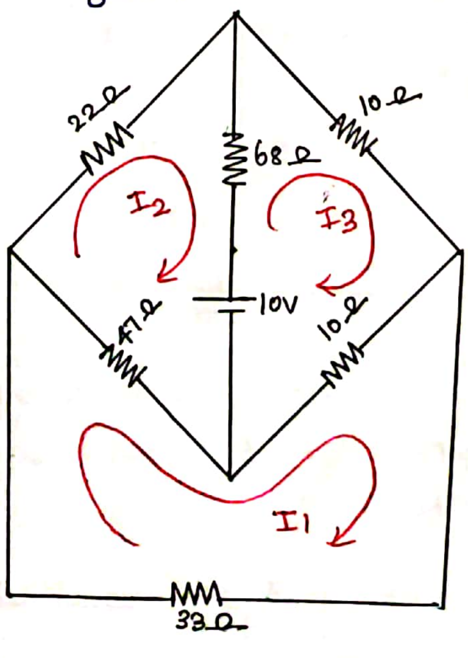
$$= \frac{20}{53.39}$$

$$I = 0.374\ A$$

consider 10V source alone
short circuit 20 V source.



using Mesh analysis.



Mesh 1 (KVL)

$47(I_1-I_2) + 10(I_1-I_3) + 33I_1 = 0$

$47I_1 - 47I_2 + 10I_1 - 10I_3 + 33I_1 = 0$

$90I_1 - 47I_2 - 10I_3 = 0$ ——— (1)

Mesh 2 (KVL)

$22I_2 + 68(I_2-I_3) + 10 + 47(I_2-I_1) = 0$

$22I_2 + 68I_2 - 68I_3 + 10 + 47I_2 - 47I_1 = 0$

$-47I_1 + 137I_2 - 68I_3 = -10$ ——— (2)

Mesh 3 (KVL)

$-10 + 68(I_3-I_2) + 10I_3 + 10(I_3-I_1) = 0$

$68I_3 - 68I_2 + 10I_3 + 10I_3 - 10I_1 = 10$

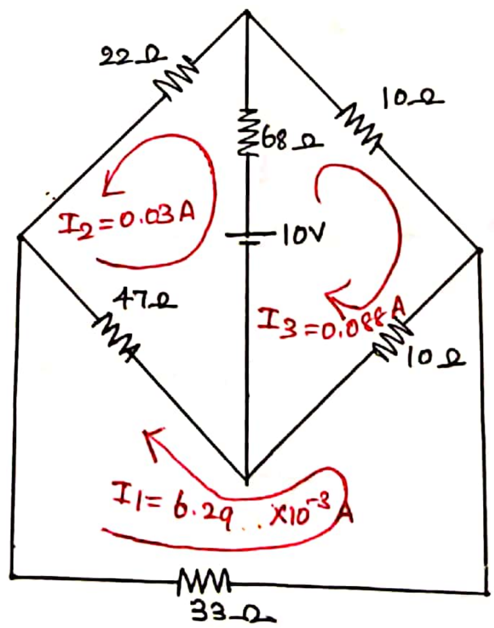
$-10I_1 - 68I_2 + 88I_3 = 10$ ——— (3)

Solving (1), (2) & (3) we get

$I_1 = -6.29.. \times 10^{-3}$ A

$I_2 = -0.03$ A

$I_3 = 0.088$ A



using superposition theorem,

source current $= 0.374 - 6.29\times10^{-3}$

$= 0.367$ A

Power delivered by 20V source $= 0.367 \times 20$

$= 7.35$ W //

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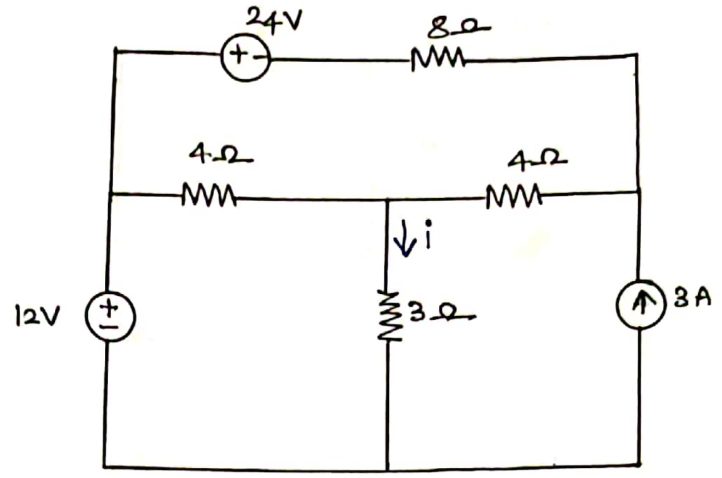
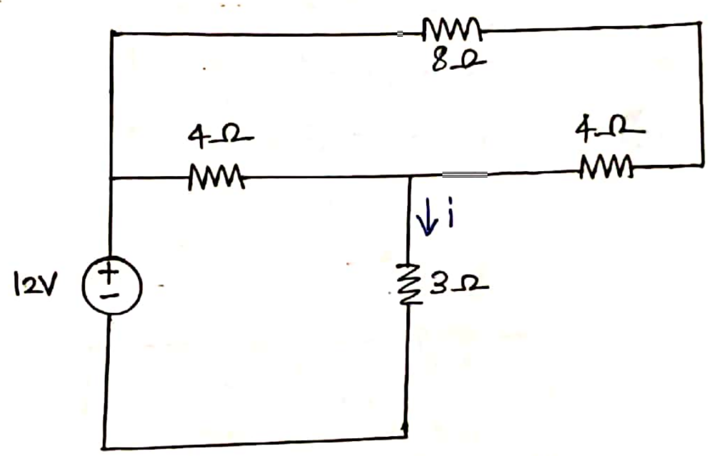
4) Apply superposition theorem to determine current i through 3Ω resistor for the given circuit in Fig.



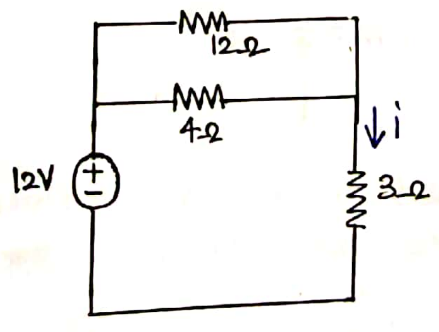
soln

Consider 12 V source alone

short circuit 24V & open circuit 3A source.



8Ω & 4Ω are in series $\Rightarrow$ $8+4 = 12\Omega$ //

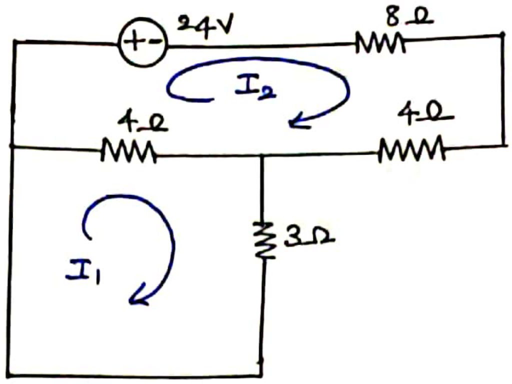


$$R_{eq} = \frac{12\times4}{12+4} + 3$$
$$= 3+3$$
$$= 6\Omega$$

$$I = V/R$$
$$= \frac{12}{6}$$
$$= 2A \;//$$

consider 24 V source alone

short circuit 12 V source and opencircuit 3 A source.



Mesh 1 (KVL)

$$4(I_1 - I_2) + 3I_1 = 0$$

$$4I_1 - 4I_2 + 3I_1 = 0$$

$$7I_1 - 4I_2 = 0 \quad \text{—— (1)}$$

Mesh 2 (KVL)

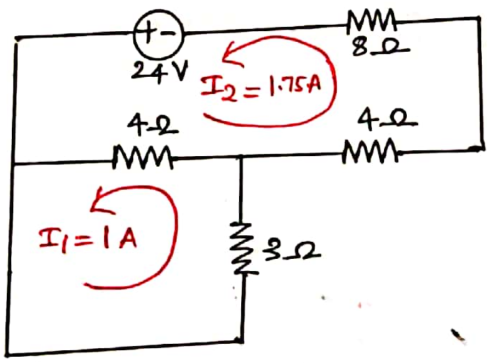
$$24 + 8I_2 + 4I_2 + 4(I_2 - I_1) = 0$$

$$8I_2 + 4I_2 + 4I_2 - 4I = -24$$

$$-4I_1 + 16I_2 = -24 \quad \text{—— (2)}$$

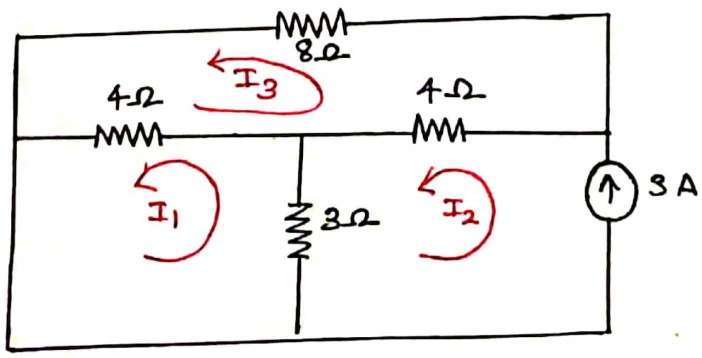
$$I_1 = -1 \text{ A}$$

$$I_2 = -1.75 \text{ A}$$



consider 3A source alone,

short circuit 24V and 12V sources



$$I_2 = 3A$$

Mesh 1 (KVL)

$$3(I_1 - I_2) + 4(I_1 - I_3) = 0$$

$$3I_1 - 3I_2 + 4I_1 - 4I_3 = 0$$

$$7I_1 - 3I_2 - 4I_3 = 0$$

$$\Rightarrow 7I_1 - 3\times 3 - 4I_3 = 0$$

$$\Rightarrow 7I_1 - 4I_3 = 9 \quad \text{—— (1)}$$

Mesh 3

$$8I_3 + 4(I_3 - I_1) + 4(I_3 - I_2) = 0$$

$$8I_3 + 4I_3 - 4I_1 + 4I_3 - 4I_2 = 0$$

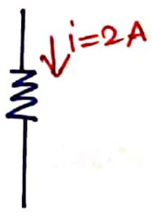
$$-4I_1 - 4I_2 + 16I_3 = 0$$

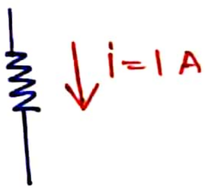
$$-4I_1 - 4\times 3 + 16I_3 = 0$$

$$-4I_1 + 16I_3 = 12 \quad \text{—— (2)}$$

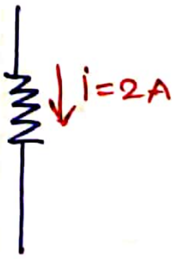
$$I_1 = 2\ A$$
$$I_3 = 1.25\ A$$

$$i = I_2 - I_1$$
$$= 3 - 2$$
$$i = 1\ A$$

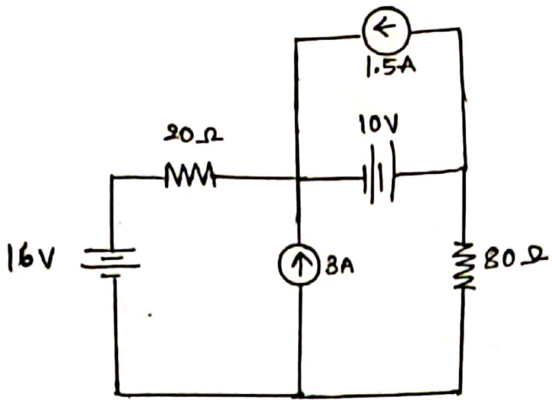




$$\therefore\ i = 2 + 1 - 1$$
$$i = 2A$$



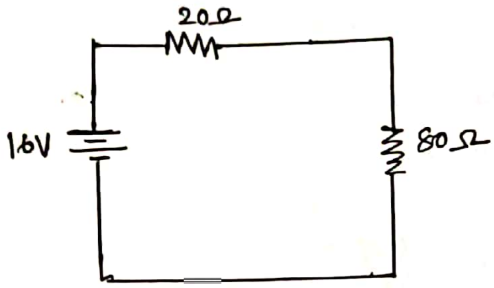
5) Determine the voltage across 20Ω resistance in the circuit shown below, using superposition theorem.

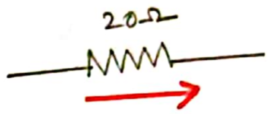


soln

Consider 16V source alone :-

open circuit 3A & 1.5A source, short ckt 10V source.





using voltage division rule

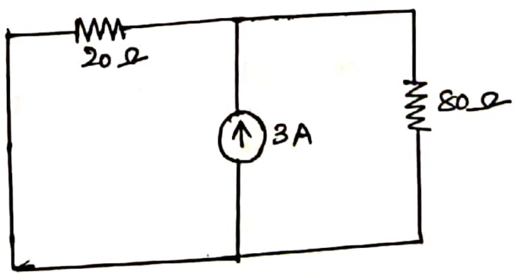
$$V_{20\Omega} = \frac{16 \times 20}{20+80}$$

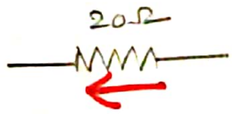
$$V_{20\Omega} = 3.2V \;//$$

Consider 3A source alone :-

short circuit 16V source & 10V source

open circuit 1.5A source.





using current division rule

$$I_{20\Omega} = \frac{3 \times 80}{20+80}$$

$$= 2.4\ A$$

$$V_{20\Omega} = 2.4 \times 20$$

$$V_{20} = 48\ V$$

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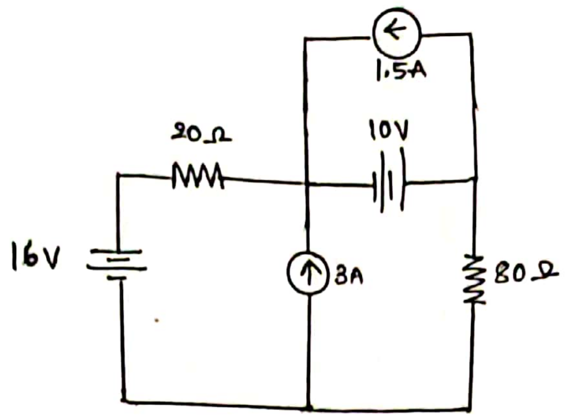
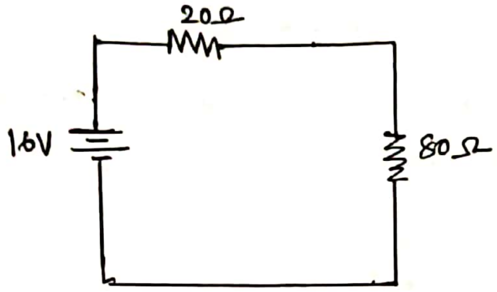
5) Determine the voltage across 20Ω resistance in the circuit shown below, using superposition theorem.

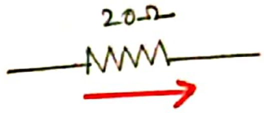


Soln

✓ consider 16V source alone :-

open circuit 3A & 1.5A source, short ckt 10V source.





using voltage division rule

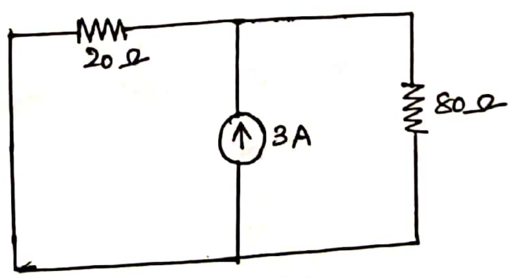
$$V_{20\Omega} = \frac{16 \times 20}{20+80}$$

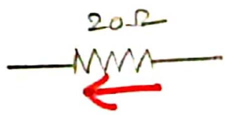
$$V_{20\Omega} = 3.2V \ //$$

✓ consider 3A source alone :-

short circuit 16V source & 10V source

open circuit 1.5A source.





using current division rule

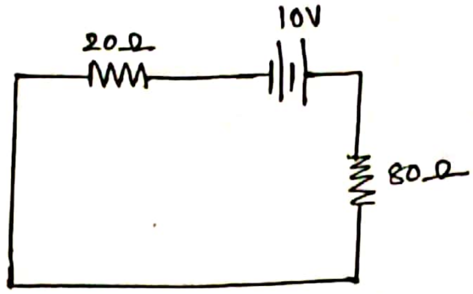
$$I_{20\Omega} = \frac{3 \times 80}{20+80}$$

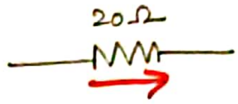
$$= 2.4 A$$

$$V_{20\Omega} = 2.4 \times 20$$

$$V_{20} = 48 V$$

✓ consider 10 V source alone
short circuit 16 V source
open circuit 3 A source and 1.5 A source.



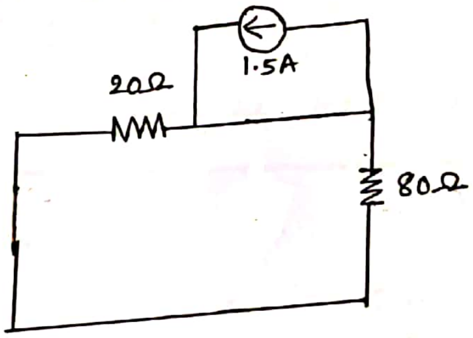


using Voltage division rules

$$V_{20\Omega} = \frac{10 \times 20}{20 + 80}$$

$$= 2\,V$$

✓ consider 1.5 A source alone
short circuit 10V source and 16 V source.
open circuit 3 A source.

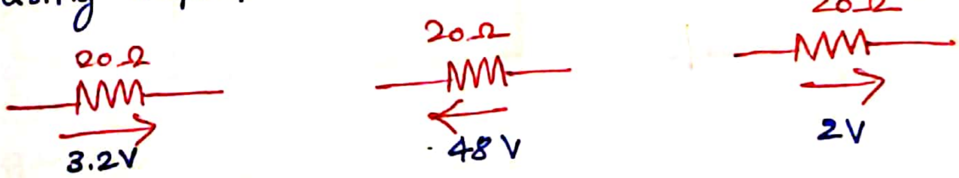


As there is direct short across 1.5A source. entire current will flow through short circuit,

$\therefore\ I_{20\Omega} = 0$

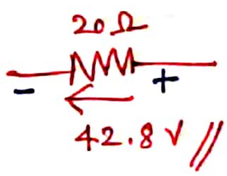
$\therefore\ V_{20\Omega} = 0$

using superposition theorem.

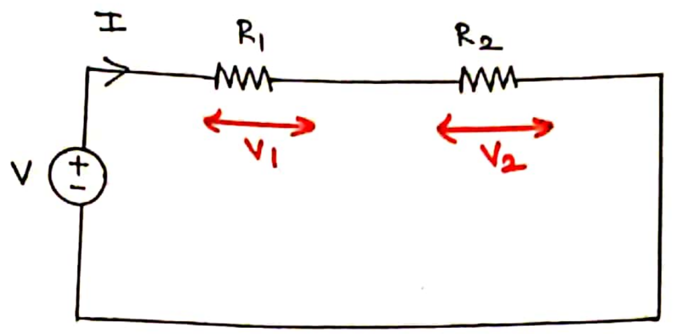


$= 48 - (3.2 + 2)$

$= 42.8\ V$



Voltage Division in series circuit of Resistors.



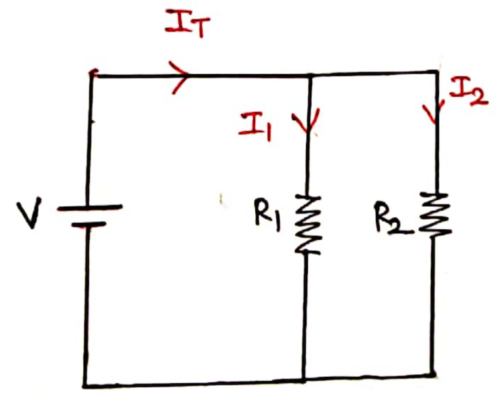
$$V_{R1} = \frac{V \cdot R_1}{R_1 + R_2}$$

$$V_{R2} = \frac{V \cdot R_2}{R_1 + R_2}$$

generally, voltage across a resistor

$\Rightarrow$ $\frac{\text{Total voltage} \times \text{value of particular resistance}}{\text{sum of resistances}}$.

Current Division rule in parallel circuit of Resistors.



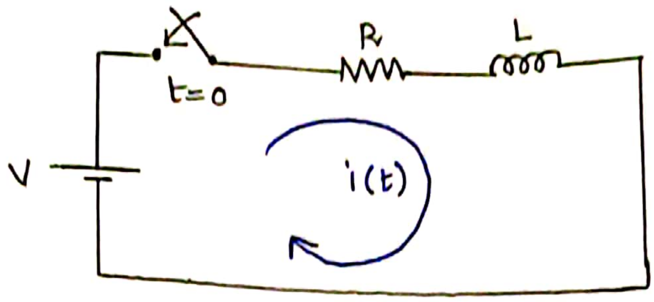
$$I_1 = \frac{I_T R_2}{R_1 + R_2}$$

$$I_2 = \frac{I_T R_1}{R_1 + R_2}$$

generally,

$\rightarrow$ $\frac{\text{Total current} \times \text{other branch resistance}}{\text{sum of resistances}}$.

RL TRANSIENTS



Apply KVL for the loop,

$$V = Ri(t) + L.\frac{di(t)}{dt}$$

Take Laplace transform.

$$\frac{V}{s} = RI(s) + L.sI(s)$$

$$\frac{V}{s} = I(s)\left[R + Ls\right]$$

$$I(s) = \frac{V}{s(R+Ls)}$$

$$= \frac{V}{Ls(s+R/L)}$$

$$I(s) = \frac{V/L}{s(s+R/L)}$$

Taking partial fraction

$$\frac{V/L}{s(s+R/L)} = \frac{A}{s} + \frac{B}{s+R/L}$$

$$\frac{V/L}{\cancel{s(s+R/L)}} = \frac{A(s+R/L) + B(s)}{\cancel{s(s+R/L)}}$$

Put $s=0$

$$\frac{V}{\cancel{L}} = A\left(\frac{R}{\cancel{L}}\right)$$

$$\boxed{A = \frac{V}{R}}$$

Put $s=-R/L$

$$\frac{V}{\cancel{L}} = B\left(-R/\cancel{L}\right)$$

$$\boxed{B = -\frac{V}{R}}$$

Now,

$$I(s) = \frac{V/R}{s} + \frac{-V/R}{s + R/L}$$

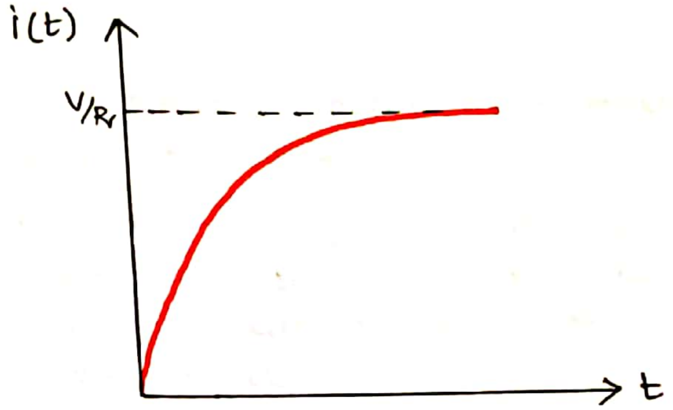
$$I(s) = \frac{V}{Rs} - \frac{V}{R(s + R/L)}$$

$$I(s) = \frac{V}{R}\left[\frac{1}{s} - \frac{1}{s + R/L}\right]$$

Take inverse Laplace transform

$$\boxed{i(t) = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)}$$

The above equation can be drawn as,



Transient voltage across resistance,

$$V_R = i(t) \cdot R$$

$$V_R = \frac{V}{\cancel{R}}\left(1 - e^{-Rt/L}\right) \cdot \cancel{R}$$

$$\boxed{V_R = V\left(1 - e^{-Rt/L}\right)}$$

Transient voltage across the inductor

$$V_L = L \cdot \frac{di(t)}{dt}$$

$$= L \cdot \frac{d}{dt} \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$= \cancel{L} \cdot \left[\frac{V}{\cancel{R}} e^{-\frac{Rt}{L}} \cdot \frac{\cancel{R}}{\cancel{L}}\right]$$

$$\boxed{V_L = V \cdot e^{-Rt/L}}$$

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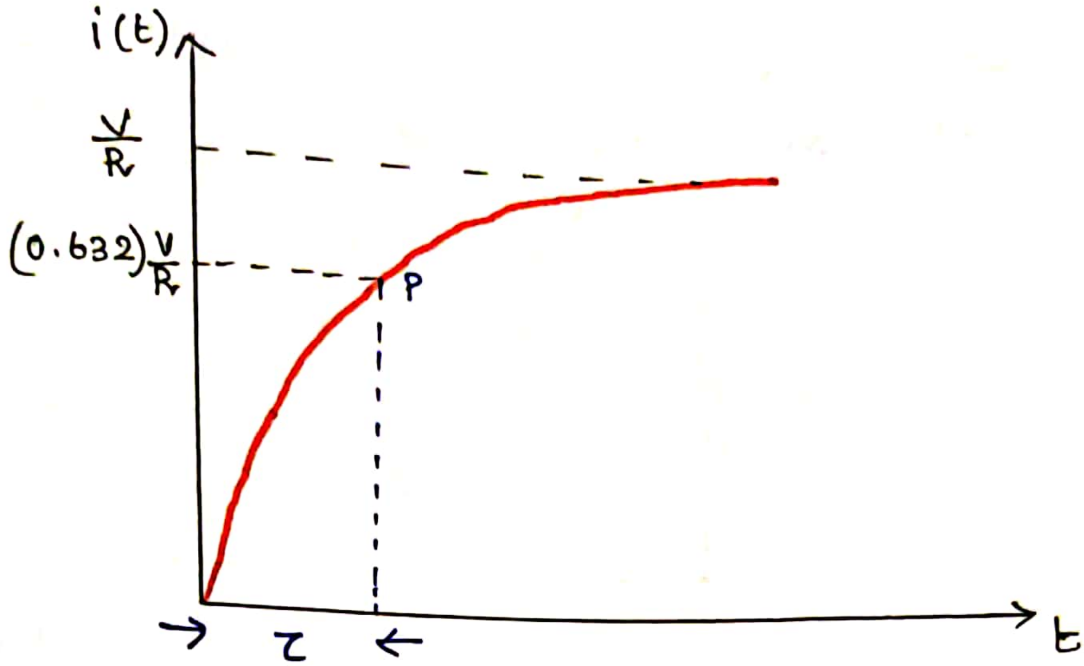
Time constant (τ) :-



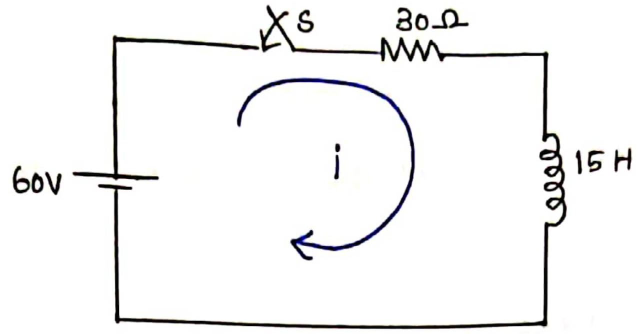
→ The current increases exponentially with respect to time.

→ The point 'p' shown in Fig. denotes the current in the circuit rises to 0.632 times maximum value of current in steady state.

→ The time required for the current to rise to the 0.632 of its final value is known as time constant of R-L circuit.

$$\boxed{\text{Time constant, } \quad \tau = \frac{L}{R}}$$

1. A series RL circuit with $R = 30\,\Omega$ and $L = 15\,H$ has a constant voltage $E = 60\,V$ applied at $t = 0$ as shown in Fig. Determine the current i, the voltage across resistor and the voltage across the inductor.



Soln,

Apply KVL

$$30i + 15\frac{di}{dt} = 60$$

$\div 15$

$$2i + \frac{di}{dt} = 4$$

Take Laplace transform.

$$2I(s) + s.I(s) = \frac{4}{s}$$

$$I(s)\left[2+s\right] = \frac{4}{s}$$

$$I(s) = \frac{4}{s(s+2)}$$

Taking partial fraction,

$$\frac{4}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$4 = A(s+2) + B(s)$$

Put $s = 0$

$2A = 4$

$\boxed{A = 2}$

Put $s = -2$

$-2B = 4$

$\boxed{B = -2}$

Now,

$$I(s) = \frac{2}{s} + \frac{-2}{s+2}$$

$$I(s) = 2\left(\frac{1}{s} - \frac{1}{s+2}\right)$$

Taking inverse Laplace transform.

$$\boxed{i(t) = 2\left(1 - e^{-2t}\right)}$$

voltage across resistor $= i(t) . R$

$$= 2\left(1-e^{-2t}\right) . 30$$

$$\boxed{V_R = 60\left(1-e^{-2t}\right)} \text{ volts.}$$

voltage across Inductor, $V_L = L.\frac{di(t)}{dt}$

$$= 15 \frac{d}{dt} 2\left(1-e^{-2t}\right)$$

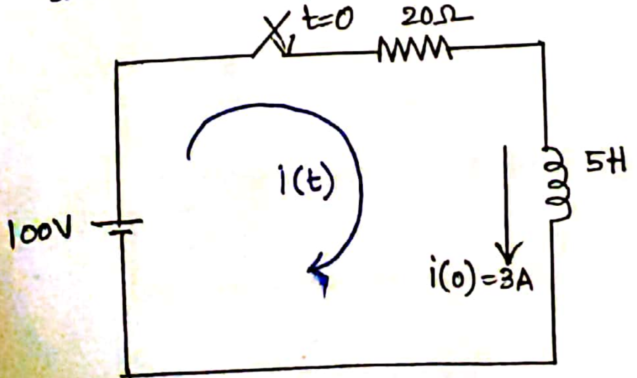
$$= 30 \frac{d}{dt}\left(1-e^{-2t}\right)$$

$$= 30\left[-e^{-2t} \times -2\right]$$

$$\boxed{V_L = 60e^{-2t}}$$

---

2) In the circuit shown in Fig. find the transient voltage across R and L after the switch is closed at time $t=0$. Assume the initial current through the inductor before the switch is closed is $3A$



Apply KVL

$$20\, i(t) + 5.\frac{di(t)}{dt} = 100$$

$\div 5$

$$4\, i(t) + \frac{di(t)}{dt} = 20$$

Take Laplace transform.

$$4I(s) + sI(s) - i(0) = \frac{20}{s}$$

$$4I(s) + sI(s) - 3 = \frac{20}{s}$$

$$I(s)\left[4+s\right] = \frac{20}{s} + 3$$

$$I(s)\left[4+s\right] = \frac{20+3s}{s}$$

$$I(s) = \frac{20+3s}{s(s+4)}$$

Taking partial fraction,

$$\frac{20+3s}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

$$20+3s = A(s+4) + B(s)$$

Put $s=0$

$$20 = 4A$$

$$\boxed{A=5}$$

Put $s=-4$

$$20+3(-4) = -4B.$$

$$20-12 = -4B$$

$$8 = -4B$$

$$\boxed{B=-2}$$

Now,

$$I(s) = \frac{5}{s} - \frac{2}{s+4}$$

Taking inverse laplace transform,

$$\boxed{i(t) = 5 - 2e^{-4t}}$$

Voltage across R,

$$V_R = i(t) \cdot R$$

$$= \left(5 - 2e^{-4t}\right) \cdot 20$$

$$\boxed{V_R = 100 - 40e^{-4t}}$$

Voltage across L,

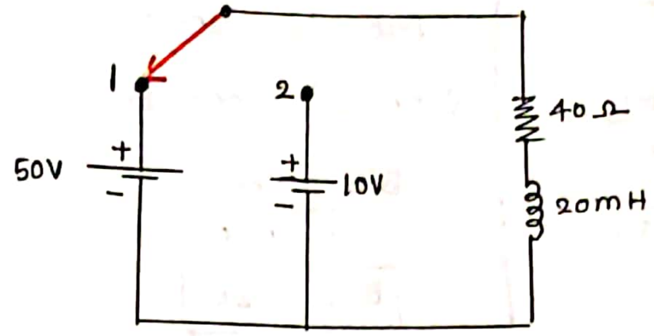
$$V_L = L \cdot \frac{di(t)}{dt}$$

$$= 5 \cdot \frac{d}{dt}\left(5 - 2e^{-4t}\right)$$

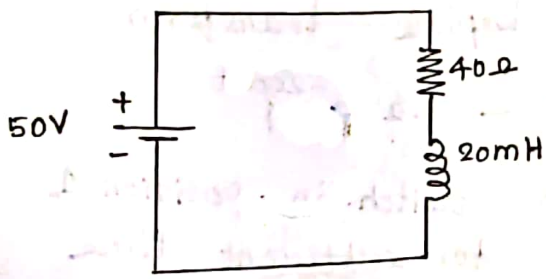
$$= 5 \cdot \left[-2e^{-4t} \cdot (-4)\right]$$

$$\boxed{V_L = 40e^{-4t}}$$

3) In the circuit as shown in Fig. the switch has been in position 1 for sufficient time to establish steady state condition. The switch is then moved to position 2. Find the current transient.



At position 1

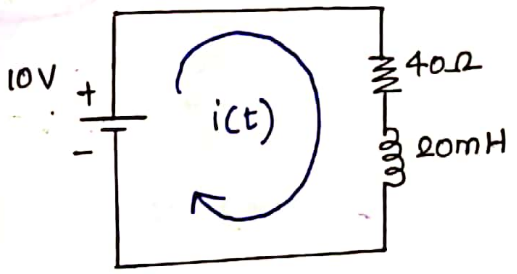


$i(0) = \frac{V}{R}$

$= \frac{50}{40}$

$= 1.25\,A$

At position 2



Apply KVL

$$40\, i(t) + [20\times10^{-3}]\frac{d\,i(t)}{dt} = 10$$

Take Laplace transform,

$$40\, I(s) + [20\times10^{-3}][s\,I(s) - i(0)] = \frac{10}{s}$$

$$40\, I(s) + [20\times10^{-3}][s\,I(s) - 1.25] = \frac{10}{s}.$$

$$40\, I(s) + (20\times10^{-3})\, s\,I(s) - 20\times10^{-3}\times1.25 = \frac{10}{s}.$$

$$I(s)\left[40 + 20\times10^{-3}\, s\right] = \frac{10}{s} + 0.025$$

$$I(s)\left[40+20\times10^{-3}s\right] = \frac{10+0.025s}{s}$$

$$I(s) = \frac{10+0.025s}{s(40+20\times10^{-3}s)}$$

$$= \frac{10+0.025s}{20\times10^{-3}s\left(\frac{40}{20\times10^{-3}}+s\right)}$$

$$= \frac{\frac{10+0.025s}{20\times10^{-3}}}{s(s+2000)}.$$

$$= \frac{\frac{10}{20\times10^{-3}}+\frac{0.025s}{20\times10^{-3}}}{s(s+2000)}.$$

$$I(s) = \frac{500+1.25s}{s(s+2000)}.$$

Taking partial fraction,

$$\frac{500+1.25s}{s(s+2000)} = \frac{A}{s}+\frac{B}{s+2000}$$

$$500+1.25s = A(s+2000)+B(s)$$

Put $s=0$

$$500 = 2000A$$

$$A = \frac{500}{2000}$$

$$\boxed{A=0.25}$$

Put $s = -2000$

$$500 + 1.25(-2000) = B(-2000)$$

$$-\cancel{2000} = B(-\cancel{2000})$$

$$\boxed{B = 1}$$

Now,

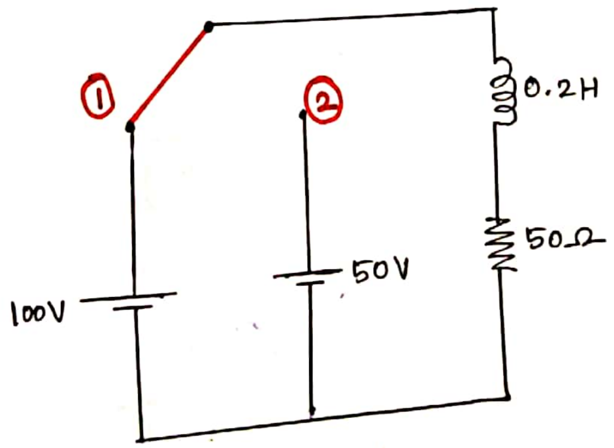
$$I(s) = \frac{0.25}{s} + \frac{1}{s+2000}$$

Take Inverse Laplace Transform.

$$\boxed{i(t) = 0.25 + e^{-2000t}}$$

---

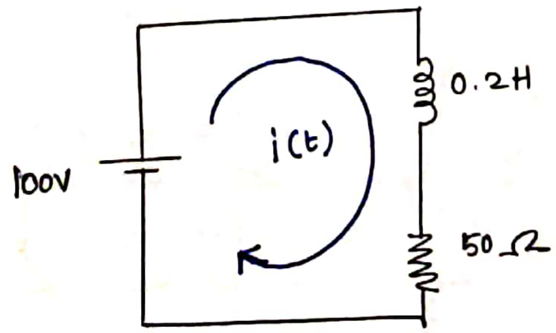
4) In the series circuit shown in Fig. the switch is closed on position 1 at $t=0$. At $t = 1$ millisecond, the switch is moved to position 2. Obtain the equations for the current in both intervals.



<u>Solution</u>

When the switch is in position 1

Apply KVL



$$50\, i(t) + 0.2 \cdot \frac{di(t)}{dt} = 100$$

Take Laplace transform,

$$50\, I(s) + 0.2\, s\, I(s) - i(0) = \frac{100}{s}.$$

$$I(s)\left[50 + 0.2s\right] = \frac{100}{s} \qquad [\because I(0) = 0]$$

$$I(s) = \frac{100}{s(50 + 0.2s)}$$

$$= \frac{\cancel{100}\ 500}{\cancel{0.2}\, s\left(s + \frac{\cancel{50}\ 250}{\cancel{0.2}}\right)}$$

$$I(s) = \frac{500}{s(s+250)}$$

Partial fraction,

$$\frac{500}{s(s+250)} = \frac{A}{s} + \frac{B}{s+250}$$

$$500 = A(s+250) + B(s)$$

Put $s = 0$

$$500 = A(250)$$

$$A = \frac{\cancel{500}\ 2}{\cancel{250}}$$

$$\boxed{A = 2}$$

Put $s = -250$

$$500 = B(-250)$$

$$\boxed{B = -2}$$

$$I(s) = \frac{2}{s} - \frac{2}{s+250}$$

$$I(s) = 2\left[\frac{1}{s} - \frac{1}{s+250}\right]$$

$$\boxed{i(t) = 2\left[1 - e^{-250t}\right]}$$

Given,

The switch is moved to position 2 at $t = 1ms = 1 \times 10^{-3}$.

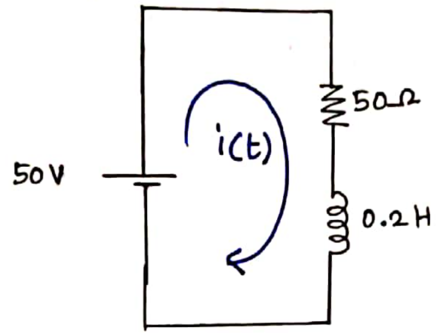
$$i(t) = 2\left[1 - e^{-250t}\right]$$

$$= 2\left[1 - e^{-250 \times 1 \times 10^{-3}}\right]$$

$$i(t) = 0.44\ A$$

For position 2 $\Rightarrow i(0) = 0.44$ A

At position 2 :-



Apply KVL

$$50 = 50\,i(t) + 0.2\frac{d\,i(t)}{dt}$$

Take Laplace Transform,

$$\frac{50}{s} = 50\,I(s) + 0.2\left[s\,I(s) - i(0)\right]$$

$$\frac{50}{s} = I(s)\left[50 + 0.2s\right] - 0.44 \times 0.2$$

$$I(s)\left[50 + 0.2s\right] = \frac{50}{s} + 0.088$$

$$I(s)\left[50 + 0.2s\right] = \frac{50 + s(0.088)}{s}$$

$$I(s) = \frac{50 + s(0.088)}{s(50 + 0.2s)}$$

$$I(s) = \frac{50 + s(0.088)}{0.2s\left(\cancel{\frac{50}{0.2}}^{250} + s\right)}$$

$$I(s) = \frac{\frac{50}{0.2} + \frac{s(0.088)}{0.2}}{s(s + 250)}$$

$$I(s) = \frac{250 + 0.44s}{s(s+250)}$$

Partial fraction,

$$\frac{250 + 0.44s}{s(s+250)} = \frac{A}{s} + \frac{B}{s+250}$$

$$250 + 0.44s = A(s+250) + B(s)$$

Put $s = 0$

$250 = 250A$

$\boxed{A = 1}$

Put $s = -250$

$250 + 0.44(-250) = B(-250)$

$140 = B(-250)$

$B = \frac{140}{-250}$

$\boxed{B = -0.56}$

Now,

$$I(s) = \frac{1}{s} - \frac{0.56}{s+250}$$

Taking inverse laplace transform.

$$\boxed{i(t) = 1 - 0.56e^{-250t}}$$

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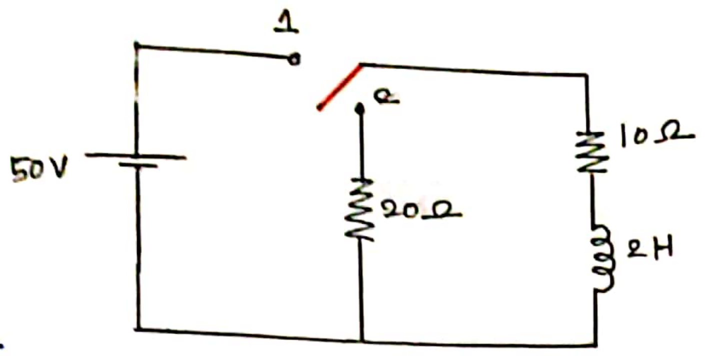
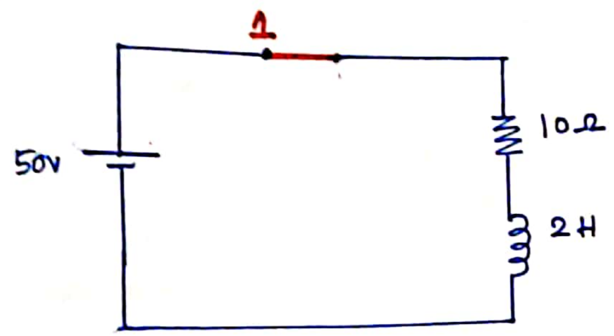
5) In the circuit shown in Fig, the switch is in position 1 for a long time and then moved to position 2. Find the transient current



Soln

Position 1



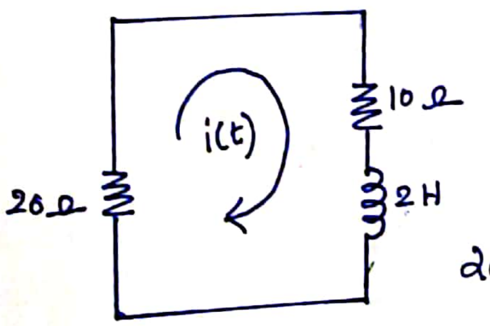
$$I = \frac{V}{R}$$

$$= \frac{50}{10}$$

$$= 5A$$

$$i(0) = 5A$$

Position 2



$$0 = 20 \cdot i(t) + 10\, i(t) + 2 \cdot \frac{di(t)}{dt}$$

Take Laplace Transform

$$20\, I(s) + 10\, I(s) + 2\left[s \cdot I(s) - i(0)\right] = 0$$

$$20 I(s) + 10 I(s) + 2s I(s) - 2 \times 5 = 0$$

$$I(s)\left[20 + 10 + 2s\right] = 10$$

$$I(s) = \frac{10}{30 + 2s}$$

$$I(s) = \frac{\cancel{10}\;5}{\cancel{2}\left(\frac{\cancel{30}\;15}{\cancel{2}}+s\right)}$$

$$I(s) = \frac{5}{15+s}$$

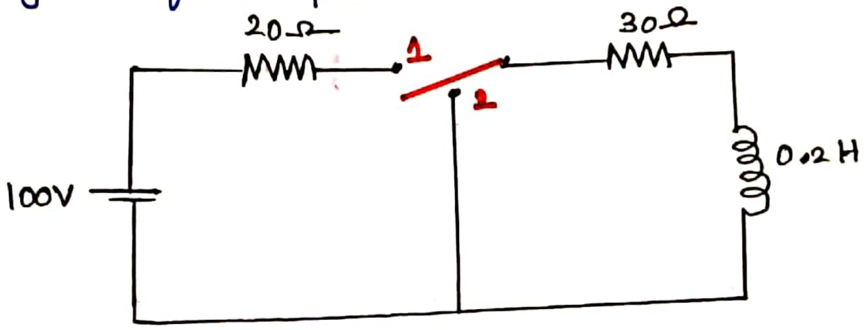
$$I(s) = \frac{5}{s+15}$$

Taking Inverse Laplace transform

$$\boxed{i(t) = 5 \cdot e^{-15t}}$$

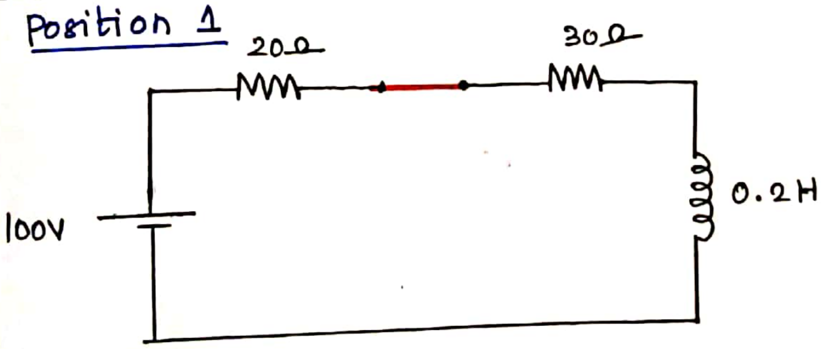
---

6. For the circuit shown in Fig, find the current equation when the switch is changed from position 1 to position 2 at $t=0$



soln

Position 1

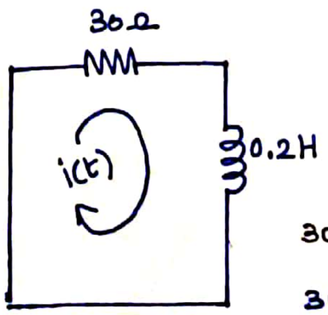


$$i(0) = \frac{V}{R}$$

$$= \frac{100}{20+30}$$

$$i(0) = 2A \,//$$

Position 2



Apply KVL

$$30\,i(t) + 0.2\frac{di(t)}{dt} = 0$$

Take Laplace transform.

$$30I(s) + 0.2\left[sI(s) - i(0)\right] = 0$$

$$30I(s) + 0.2\left[sI(s) - 2\right] = 0$$

$$30I(s) + 0.2sI(s) - 0.4 = 0$$

$$I(s)\left[30 + 0.2s\right] = 0.4$$

$$I(s) = \frac{0.4}{30 + 0.2s}$$

$$I(s) = \frac{0.\cancel{4}\,2}{0.\cancel{2}\left(\frac{\cancel{30}\,150}{\cancel{0.2}} + s\right)}$$

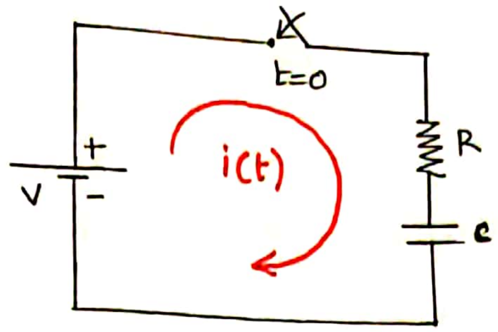
$$I(s) = \frac{2}{s + 150}$$

$$\boxed{I(t) = 2e^{-150t}}\ \text{A}$$

LAPLACE TRANSFORM OF SOME IMPORTANT FUNCTIONS

$f(t)$	$\leftrightarrow$	$f(s)$
K (constant)	$\leftrightarrow$	$\frac{K}{s}$
1	$\leftrightarrow$	$\frac{1}{s}$
t	$\leftrightarrow$	$\frac{1}{s^2}$
t^2	$\leftrightarrow$	$\frac{2}{s^3}$
t^n	$\leftrightarrow$	$\frac{n!}{s^{n+1}}$
e^{-at}	$\leftrightarrow$	$\frac{1}{s+a}$
e^{at}	$\leftrightarrow$	$\frac{1}{s-a}$
$e^{-at}\ t^n$	$\leftrightarrow$	$\frac{n!}{(s+a)^{n+1}}$
$\sin\omega t$	$\leftrightarrow$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\leftrightarrow$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\ \sin\omega t$	$\leftrightarrow$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\sinh\omega t$	$\leftrightarrow$	$\frac{\omega}{s^2-\omega^2}$
$\cosh\omega t$	$\leftrightarrow$	$\frac{s}{s^2-\omega^2}$
te^{-at}	$\leftrightarrow$	$\frac{1}{(s+a)^2}$
$1-e^{-at}$	$\leftrightarrow$	$\frac{a}{s(s+a)}$

RC TRANSIENT



Apply KVL

$$R\,i(t) + \frac{1}{C}\int i(t)\,dt = V$$

Taking Laplace Transform,

$$R\,I(s) + \frac{1}{C}\frac{1}{s}I(s) = \frac{V}{s}$$

$$I(s)\left[R + \frac{1}{Cs}\right] = \frac{V}{s}.$$

$$I(s) = \frac{V}{s\left[R + \frac{1}{Cs}\right]}$$

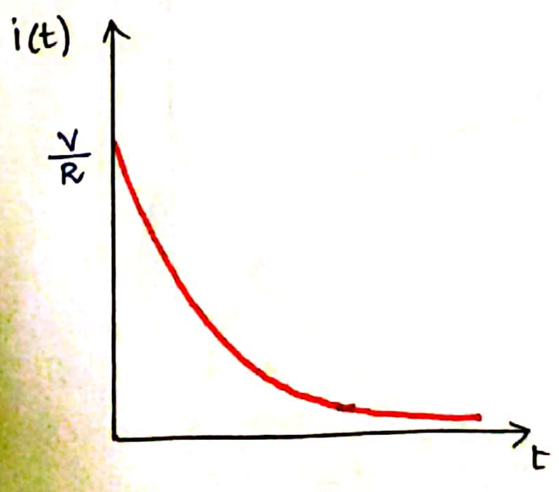
$$= \frac{V}{sR + \frac{\cancel{s}}{C\cancel{s}}}$$

$$I(s) = \frac{V}{sR + \frac{1}{C}} = \frac{V}{R\left(s + \frac{1}{RC}\right)}$$

$$\therefore\ I(s) = \frac{\frac{V}{R}}{s + \frac{1}{RC}}$$

Taking inverse Laplace Transform

$$\boxed{i(t) = \frac{V}{R}e^{-t/RC}}$$



$$i(t)\big|_{t=0} = \frac{V}{R}$$

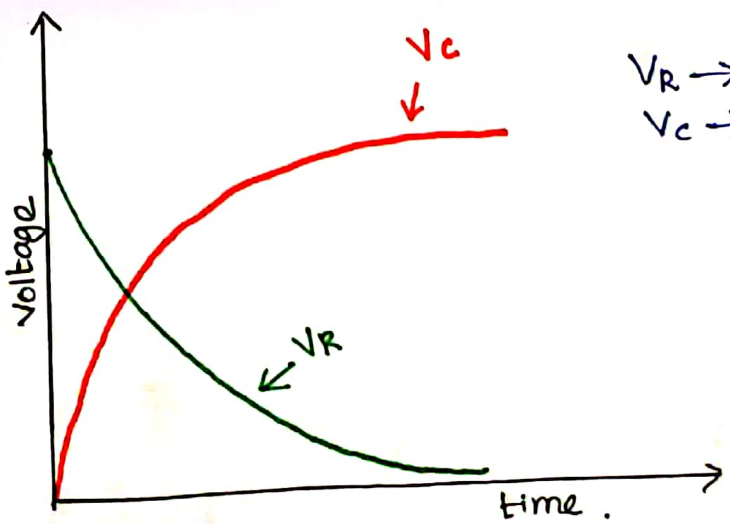
$$i(t)\big|_{t=\infty} = 0$$

The Voltage across the resistor,

$$V_R = R\,i(t)$$
$$= \cancel{R} \cdot \frac{V}{\cancel{R}} e^{-t/RC}$$
$$\boxed{V_R = V e^{-t/RC}}$$

The voltage across the capacitor,

$$V_c = \frac{1}{C}\int_0^t i(t)\,.dt$$
$$= \frac{1}{C}\int_0^t \frac{V}{R} e^{-t/RC}$$
$$= \frac{1}{C}\cdot\frac{V}{R}\int_0^t e^{-t/RC}$$
$$= \frac{V}{RC}\left[\frac{e^{-t/RC}}{-1/RC}\right]_0^t$$
$$= \frac{V}{RC}\left[-RC\,.\,e^{-t/RC}\right]_0^t$$
$$= \frac{V}{RC}\left[-RC\, e^{-t/RC} + RC\right]$$
$$= \frac{\cancel{RC}\,V}{\cancel{RC}}\left[-e^{-t/RC} + 1\right]$$
$$\boxed{V_c = V\left[1 - e^{-t/RC}\right]}$$



$V_R \rightarrow$ Decaying
$V_c \rightarrow$ rising

TIME CONSTANT FOR SERIES RC CIRCUIT.

W.K.T

$$i(t) = \frac{V}{R} e^{-t/RC} \quad \& \quad V_c = V\left[1 - e^{-t/RC}\right]$$

Time constant is obtained by putting $t = RC$

$$i(t) = \frac{V}{R} e^{-t/RC}$$

$$i(t) = \frac{V}{R} e^{-1}$$

$$i(t) = 0.3678 \frac{V}{R}$$

Hence,

time constant of RC series circuit is defined as the period during which the current decreases to 36.8 % of its initial value

$$V_c = V\left[1 - e^{-t/RC}\right]$$

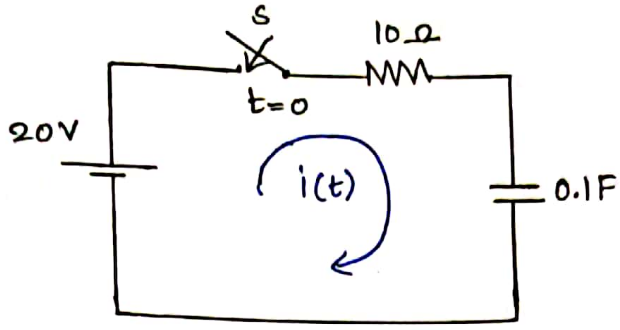
$$V_c = V\left[1 - e^{-1}\right]$$

$$V_c = 0.6321 V$$

Hence,

time constant is defined as the period during which the capacitor attain 63.2 % of steady state voltage.

1. A series RC circuit consists of a resistor of 10Ω and a capacitor of 0.1 F as shown in Fig. A constant voltage of 20V is applied to the circuit at $t=0$. Obtain the current equation. Determine the voltage across the resistor and capacitor.



solution:

Apply KVL,

$$20 = 10\, i(t) + \frac{1}{0.1}\int i(t)\,dt$$

Taking Laplace Transform,

$$10I(s) + \frac{1}{0.1}\frac{1}{s}I(s) = \frac{20}{s}.$$

$$I(s)\left[10 + \frac{1}{0.1s}\right] = \frac{20}{s}.$$

$$I(s) = \frac{20}{s\left(10+\frac{1}{0.1s}\right)}$$

$$= \frac{20}{\cancel{s}\left(\frac{10\times 0.1s + 1}{0.1\cancel{s}}\right)}$$

$$= \frac{20}{\frac{s+1}{0.1}}$$

$$= \frac{20\times 0.1}{s+1}$$

$$I(s) = \frac{2}{s+1}$$

Taking Inverse Laplace transform

$$\boxed{i(t) = 2e^{-t}}$$

Voltage across the resistor,

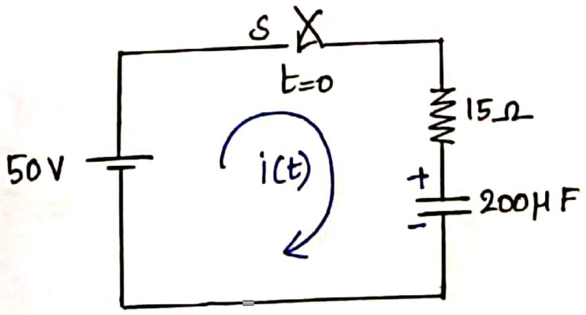
$$V_R = i(t) \times R$$
$$= 2e^{-t} \times 10$$
$$\boxed{V_R = 20e^{-t}}$$

Voltage across the capacitor,

$$V_C = \frac{1}{C}\int i(t).dt$$
$$= \frac{1}{0.1}\int_0^t 2e^{-t}.dt$$
$$= \frac{2}{0.1}\int_0^t e^{-t}.dt$$
$$= 20\left[\frac{e^{-t}}{-1}\right]_0^t$$
$$= 20\left[-e^{-t}\right]_0^t$$
$$= 20\left[-e^{-t} + e^0\right]$$
$$= 20\left[-e^{-t}+1\right]$$
$$\boxed{V_C = 20\left[1-e^{-t}\right]}$$

---

2)(i) In the circuit shown in Fig. Find the transient current after the switch is closed at t=0. An initial charge of 100μC is stored in the capacitor as shown.



solution

Apply KVL

$$15\,i(t) + \frac{1}{200\times10^{-6}}\int i(t).dt + V_0 = 50$$

Initial charge of capacitor, $V_0 = \frac{Q}{C} = \frac{100\times10^{-6}}{200\times10^{-6}}$

$V_0 = 0.5$

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$$15\,i(t) + 5000\int i(t).dt + 0.5 = 50$$

$$15\,i(t) + 5000\int i(t).dt = 49.5$$

Take Laplace transform.

$$15\,I(s) + 5000.\frac{1}{s}.I(s) = \frac{49.5}{s}$$

$$I(s)\left[15 + \frac{5000}{s}\right] = \frac{49.5}{s}.$$

$$I(s)\left[\frac{15s + 5000}{\cancel{s}}\right] = \frac{49.5}{\cancel{s}}$$

$$I(s) = \frac{49.5}{15s + 5000}$$

$$= \frac{49.5}{15\left(s + \frac{5000}{15}\right)}$$

$$= \frac{49.5/15}{(s + 333.33)}$$

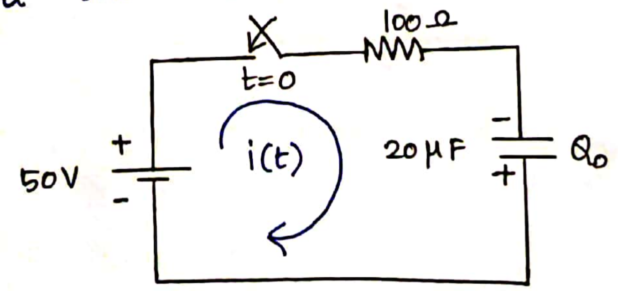
$$I(s) = \frac{3.3}{(s + 333.33)}$$

Take Inverse Laplace transform.

$$\boxed{i(t) = 3.3e^{-333.33t}}$$

---

2)(ii) The 20μF capacitor in circuit of figure has an initial charge $Q_0 = 0.0001$ Coulomb as shown. The switch is closed at $t=0$. Find the transient current.



Apply KVL

$$100\,i(t) + \frac{1}{20\times10^{-6}}\int i(t).dt - V_0 = 50$$

$$V_0 = \frac{Q}{C}$$

$$= \frac{0.001}{20\times10^{-6}}$$

$$V_0 = 50\text{ V}$$

$$100\,i(t) + 50000\int i(t).dt - 50 = 50$$

$$100\,i(t) + 50000\int i(t).dt = 100$$

$\div 100$

$$i(t) + 500\int i(t).dt = 1$$

Take Laplace transform.

$$I(s) + 500\,\frac{1}{s}\,.\,I(s) = \frac{1}{s}$$

$$I(s)\left[1 + \frac{500}{s}\right] = \frac{1}{s}$$

$$I(s)\left[\frac{s+500}{s}\right] = \frac{1}{s}$$

$$I(s) = \frac{1}{\cancel{s}} \times \frac{\cancel{s}}{s+500}$$

$$I(s) = \frac{1}{s+500}$$

Take Inverse Laplace transform,

$$\boxed{i(t) = e^{-500t}}$$

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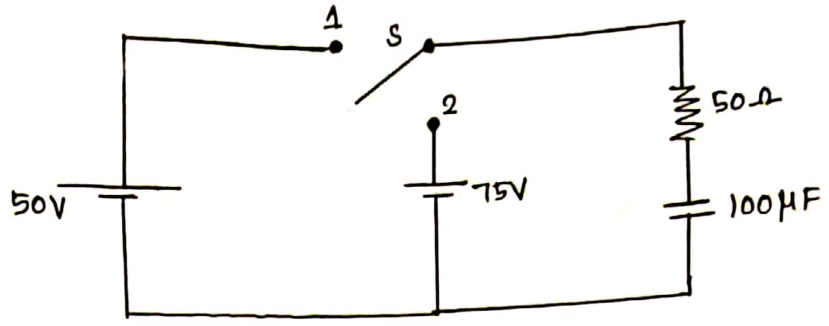
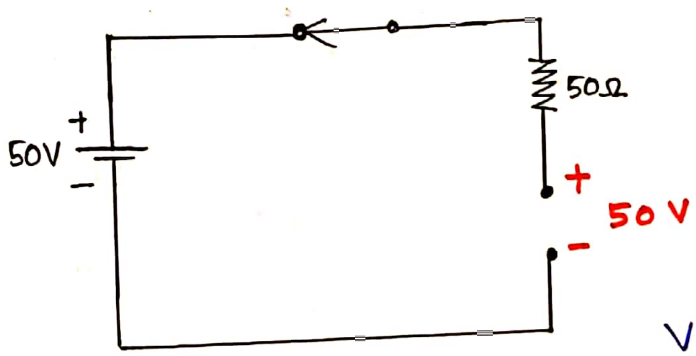
3) In the circuit shown in Fig, the switch is put in position 1 for a long time and then thrown to position 2. Find the transient current assuming no initial charge on capacitor.



solution 1

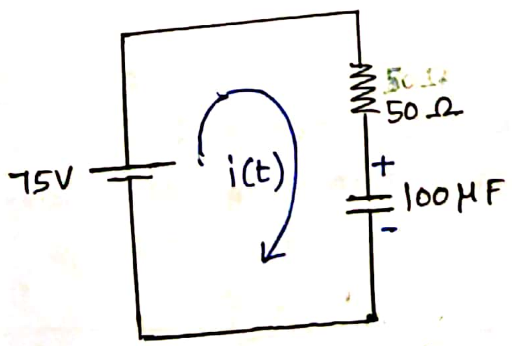
When switch In position 1

The network is in steady state so capacitor acts as open circuit.



$V_0 = 50V$

When put in position 2



$$50\, i(t) + \frac{1}{100\times10^{-6}} \int i(t).dt + V_0 = 75$$

$$50\, i(t) + \frac{1}{100\times10^{-6}} \int i(t).dt + 50 = 75$$

$$50\,i(t) + 10000\int_0^t i(t)\,.\,dt = 25\,.$$

Take Laplace transform.

$$50\,I(s) + 10000\,\frac{1}{s}\,I(s) = \frac{25}{s}.$$

$$I(s)\left[50 + \frac{10000}{s}\right] = \frac{25}{s}.$$

$$I(s)\left[\frac{50s + 10000}{\cancel{s}}\right] = \frac{25}{\cancel{s}}$$

$$I(s) = \frac{25}{50s + 10000}$$

$$= \frac{25}{50\left[s + \frac{10000}{50}\right]}$$

$$= \frac{25}{50\left[s + 200\right]}$$

$$I(s) = \frac{1}{2(s+200)}$$

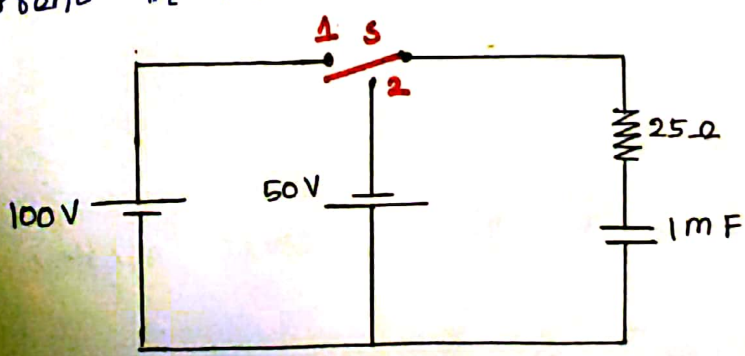
$$I(s) = \frac{0.5}{s+200}$$

Take Inverse Laplace transform

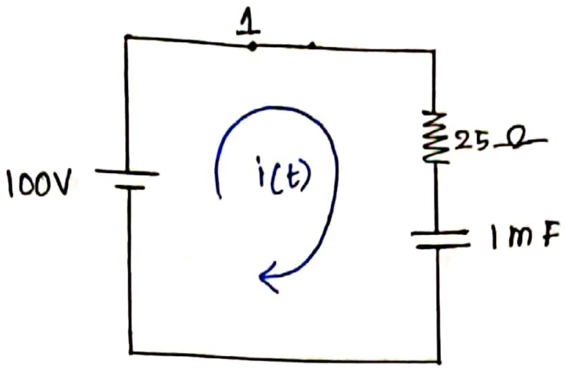
$$\boxed{i(t) = 0.5e^{-200t}}$$

---

4) In the circuit shown in Fig. the switch is in position 1 for 1 ms and then thrown to position 2. Find the transient current in both intervals.



When switch is in position 1



$$25\, i(t) + \frac{1}{1\times10^{-3}} \int i(t)\,dt = 100$$

Take Laplace transform.

$$25\, I(s) + 1000 \cdot \frac{1}{s} \cdot I(s) = \frac{100}{s}$$

$$I(s)\left[25 + \frac{1000}{s}\right] = \frac{100}{s}$$

$$I(s)\left[\frac{25s + 1000}{\cancel{s}}\right] = \frac{100}{\cancel{s}}$$

$$I(s) = \frac{100}{25s + 1000}$$

$$I(s) = \frac{\cancel{100}\ 4}{\cancel{25}\left(s + \frac{1000}{25}\right)}$$

$$I(s) = \frac{4}{s + 40}$$

Take Inverse Laplace transform.

$$i(t) = 4e^{-40t}$$

$$\text{Voltage across capacitor} = \frac{1}{C}\int i(t)\,dt$$

$$= \frac{1}{1\times10^{-3}} \int_0^t 4e^{-40t}\,dt$$

$$= \frac{4}{1\times10^{-3}} \int_0^t e^{-40t}\,.dt$$

$$= \frac{4}{1\times10^{-3}} \left[\frac{e^{-40t}}{-40} \right]_0^t$$

$$= \frac{\cancel{4}}{1\times10^{-3} \times -\underset{10}{\cancel{40}}} \left[e^{-40t} \right]_0^t$$

$$= -100 \left[e^{-40t} - 1 \right]$$

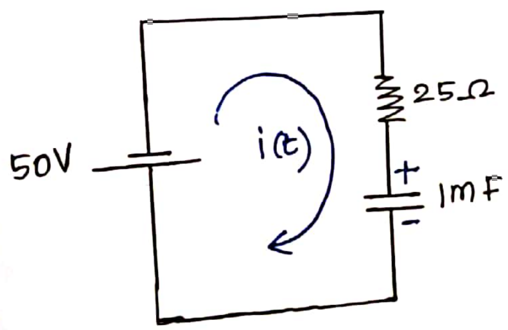
$$V_c = 100 - 100e^{-40t}$$

at $t = 1ms$

$$V_c = 100 - 100e^{-40\times(1\times10^{-3})}$$

$$\boxed{V_c = 3.921\ V} \Rightarrow V_0$$

Position 2



$$50 + 25\,i(t) + \frac{1}{1\times10^{-3}} \int i(t)\,.dt + V_0 = 0$$

$$50 + 25\,i(t) + 1000 \int i(t)\,.dt + 3.921 = 0$$

$$25\,i(t) + 1000 \int i(t)\,.dt = -53.921$$

Taking Laplace transform,

$$25I(s) + 1000\frac{1}{s}\,.\,I(s) = \frac{-53.921}{s}$$

$$I(s)\left[25 + \frac{1000}{s}\right] = -\frac{53.921}{s}$$

$$I(s)\left[\frac{25s+1000}{\cancel{s}}\right] = \frac{-53.921}{\cancel{s}}.$$

$$I(s) = \frac{-53.921}{25s+1000}$$

$$I(s) = \frac{-53.921}{25\left(s+\frac{1000}{25}\right)}$$

$$I(s) = \frac{-53.921}{25(s+40)}$$

$$I(s) = \frac{-2.156}{s+40}$$

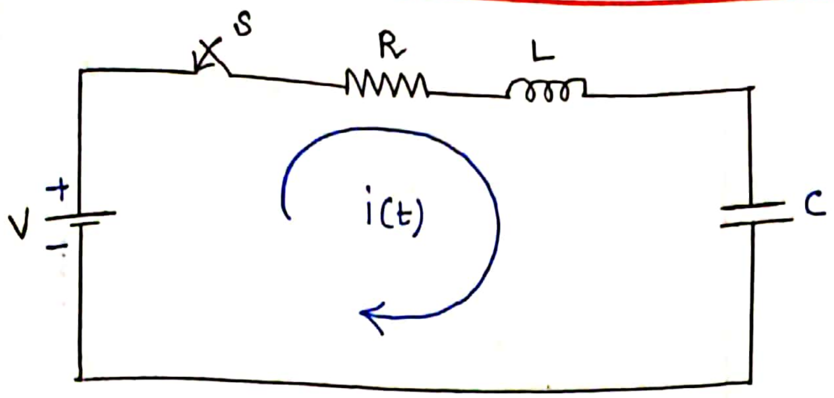
Take Inverse Laplace

$$i(t) = -2.156\,e^{-40t}$$

Therefore

$$\boxed{\begin{array}{lll} i(t) = 4e^{-40t} & \text{for} & 0 < t < 1\,ms \\ i(t) = -2.156e^{-40t} & \text{for} & t > 0.001\,ms. \end{array}}$$

TRANSIENTS IN RLC SERIES CIRCUIT



Apply KVL

$$R\,i(t) + L.\frac{di(t)}{dt} + \frac{1}{C}\int i(t).dt = V$$

Taking Laplace transform

$$RI(s) + L.s.I(s) + \frac{1}{C}.\frac{1}{s}\,I(s) = \frac{V}{s}$$

$$I(s)\left[R + Ls + \frac{1}{Cs}\right] = \frac{V}{s}.$$

$$I(s) = \frac{V}{s\left[R + Ls + \frac{1}{Cs}\right]}$$

$$= \frac{V}{sR + Ls^2 + \frac{\cancel{s}}{C\cancel{s}}}$$

$$I(s) = \frac{V}{Ls^2 + Rs + \frac{1}{C}}$$

$$I(s) = \frac{V}{L\left(s^2 + \frac{Rs}{L} + \frac{1}{CL}\right)}$$

$$I(s) = \frac{V/L}{s^2 + \frac{Rs}{L} + \frac{1}{CL}}$$

$$s^2 + \frac{Rs}{L} + \frac{1}{CL}$$

$$s = \frac{\frac{-R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(\frac{1}{CL}\right)}}{2}$$

$$s = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \cancel{4}\frac{1}{\cancel{4}CL}}$$

$$s = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{CL}}$$

$$s = \alpha \pm \beta$$

where $\alpha = \frac{-R}{2L}$, $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{CL}}$

For understanding only.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$

$b = \frac{R}{L}$

$c = \frac{1}{CL}$

case (i)

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

The two roots are real and distinct

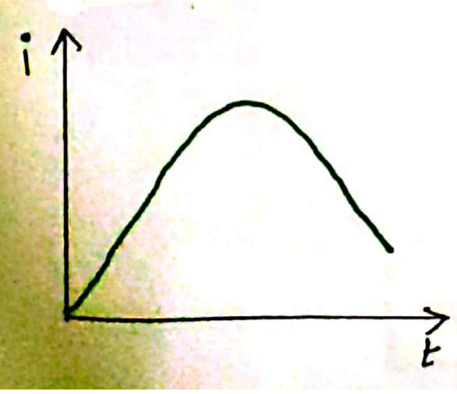
$$I(s) = \frac{K_1}{s-(\alpha+\beta)} + \frac{K_2}{s-(\alpha-\beta)}$$

$$i(t) = K_1 e^{(\alpha+\beta)t} + K_2 e^{(\alpha-\beta)t}$$

$$= K_1 e^{\alpha t} \cdot e^{\beta t} + K_2 e^{\alpha t} \cdot e^{-\beta t}$$

$$i(t) = e^{\alpha t}\left[K_1 e^{\beta t} + K_2 e^{-\beta t}\right]$$

The current is over damped.



case (ii)

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{CL}$$

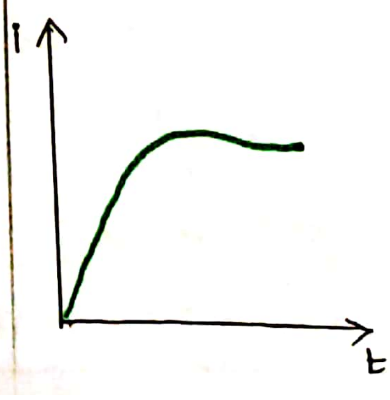
The roots are real and equal

$$I(s) = \frac{K_1}{(s-\alpha)^2} + \frac{K_2}{(s-\alpha)}$$

$$i(t) = K_1 t e^{\alpha t} + K_2 e^{\alpha t}$$

$$i(t) = e^{\alpha t}\left[K_1 t + K_2\right]$$

The current is critically damped



case (iii)

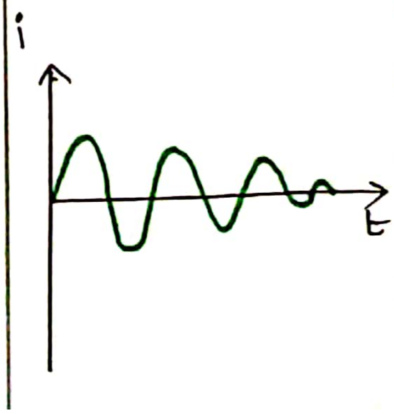
$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

The roots become complex conjugate

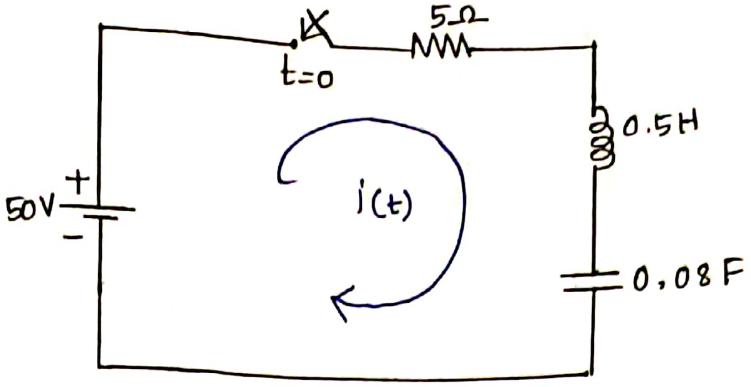
$$I(s) = \frac{K_1}{s-(\alpha+j\beta)} + \frac{K_2}{s-(\alpha-j\beta)}$$

$$i(t) = e^{\alpha t}\left[K_1 e^{j\beta t} + K_2 e^{-j\beta t}\right]$$

The current is oscillatory in nature.



1. In the circuit shown in Fig, Find the transient current when the switch is closed at $t=0$. Assume zero initial conditions.



solution:-

Apply KVL

$$5i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{0.08}\int i(t).dt = 50$$

Take Laplace transform,

$$5I(s) + 0.5s\,I(s) + \frac{1}{0.08}\cdot\frac{1}{s}\cdot I(s) = \frac{50}{s}$$

$$I(s)\left[5 + 0.5s + \frac{1}{0.08s}\right] = \frac{50}{s}$$

$$I(s) = \frac{50}{s\left[5 + 0.5s + \frac{1}{0.08s}\right]}$$

$$= \frac{50}{5s + 0.5s^2 + \frac{1}{0.08}}$$

$$= \frac{50}{0.5\left[s^2 + \frac{5s}{0.5} + \frac{12.5}{0.5}\right]}$$

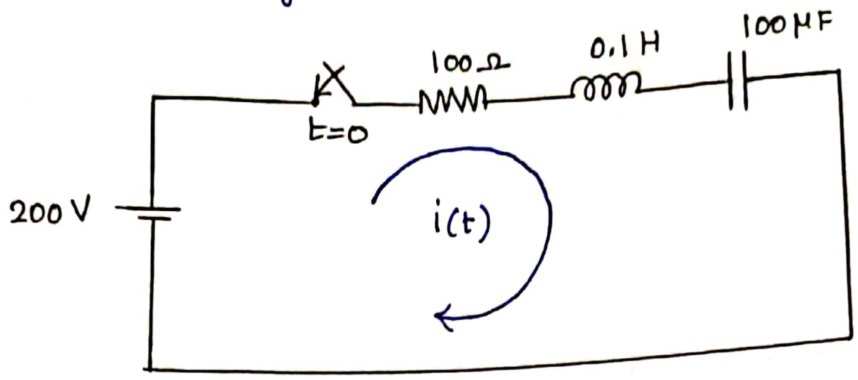
$$= \frac{50/0.5}{s^2 + 10s + 25}$$

$$I(s) = \frac{100}{(s+5)^2}$$

Take Inverse Laplace transform

$$\boxed{i(t) = 100\,t e^{-5t}}$$

2) A series RLC circuit with $R = 100\,\Omega$, $L = 0.1\,H$ and $C = 100\,\mu F$ has a DC voltage of 200 volts applied to it at $t=0$ through a switch. Find the expression for the transient current. Assume initially relaxed circuit conditions.



solution,

Apply

$$100\, i(t) + 0.1 \frac{d\,i(t)}{dt} + \frac{1}{100\times10^{-6}} \int i(t)\,.dt = 200$$

Take Laplace Transform,

$$100\, I(s) + 0.1\, s\, I(s) + 10000 \cdot \frac{1}{s} \cdot I(s) = \frac{200}{s}$$

$$I(s)\left[100 + 0.1s + \frac{10000}{s}\right] = \frac{200}{s}$$

$$I(s) = \frac{200}{s\left[100 + 0.1s + \frac{10000}{s}\right]}$$

$$= \frac{200}{100s + 0.1s^2 + 10000}$$

$$= \frac{200}{0.1\left[s^2 + \frac{100s}{0.1} + \frac{10000}{0.1}\right]}$$

$$= \frac{200/0.1}{s^2 + 1000s + 100000}$$

$$I(s) = \frac{2000}{s^2 + 1000s + 100000}$$

Dr

$$s^2 + 1000s + 100000 = 0$$

$$\boxed{s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$s = \frac{-1000 \pm \sqrt{(1000)^2 - 4 \times 1 \times 100000}}{2}$$

$$= \frac{-1000 \pm 774.596}{2}$$

$$= \frac{-1000 + 774.596}{2}$$

$$s = -112.702$$

$$\frac{-1000 - 774.596}{2}$$

$$s = -887.298$$

Now,

$$I(s) = \frac{K_1}{s - (-112.702)} + \frac{K_2}{s - (-887.298)}$$

$$\frac{2000}{s^2 + 1000s + 100000} = \frac{K_1}{s + 112.702} + \frac{K_2}{s + 887.298}$$

$$\frac{2000}{\cancel{s^2 + 1000s + 100000}} = \frac{K_1(s + 887.298) + K_2(s + 112.702)}{\cancel{(s + 112.702)(s + 887.298)}}$$

$$2000 = K_1(s + 887.298) + K_2(s + 112.702)$$

When $s = -887.298$

$$2000 = K_2(-887.298 + 112.702)$$

$$2000 = K_2(-774.576)$$

$$K_2 = \frac{2000}{-774.576}$$

$$\boxed{K_2 = -2.58}$$

When $s = -112.702$

$$2000 = K_1(-112.702 + 887.298)$$

$$2000 = K_1(774.596)$$

$$K_1 = \frac{2000}{774.596}$$

$$\boxed{K_1 = 2.58}$$

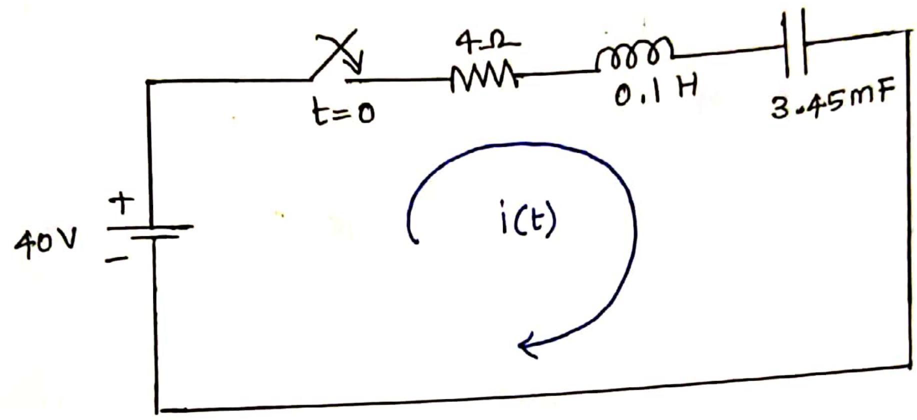
$$I(s) = \frac{2.58}{s+112.702} - \frac{2.58}{s+887.298}$$

$$= 2.58\left[\frac{1}{s+112.702} - \frac{1}{s+887.298}\right]$$

$$\boxed{i(t) = 2.58\left[e^{-112.702t} - e^{-887.298t}\right]}$$

3) In the circuit shown in Fig. Find the transient current when the switch is closed at $t=0$.

Assume zero Initial conditions.



solution:

Apply KVL

$$4\,i(t) + 0.1\frac{di(t)}{dt} + \frac{1}{3.45\times10^{-3}}\int i(t).dt = 40$$

Take Laplace transform.

$$4I(s) + 0.1sI(s) + \frac{1}{3.45\times10^{-3}}\cdot\frac{1}{s}I(s) = \frac{40}{s}$$

$$I(s)\left[4 + 0.1s + \frac{1}{3.45\times10^{-3}}\cdot\frac{1}{s}\right] = \frac{40}{s}$$

$$I(s)\left[4 + 0.1s + \frac{289.85}{s}\right] = \frac{40}{s.}$$

$$I(s) = \frac{40}{s\left[4 + 0.1s + \frac{289.85}{s}\right]}$$

$$I(s) = \frac{40}{4s + 0.1s^2 + 289.85}$$

$$I(s) = \frac{40}{0.1\left[s^2 + \frac{4s}{0.1} + \frac{289.85}{0.1}\right]}$$

$$= \frac{40/0.1}{s^2 + 40s + 2898.5}$$

$$I(s) = \frac{400}{s^2 + 40s + 2898.5}.$$

DS

$$s^2 + 40s + 2898.5$$

$$\boxed{s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$a = 1$
$b = 40$
$c = 2898.5$

$$s = \frac{-40 \pm \sqrt{40^2 - 4 \times 2898.5}}{2}$$

$$s = \frac{-40 \pm \sqrt{1600 - 11594}}{2}$$

$$s = \frac{-40 \pm \sqrt{-9994}}{2}$$

$$s = \frac{-40 \pm 99.9i}{2}$$

$$s = -20 \pm 50i$$

$\alpha = -20$
$\beta = 50$

The roots are complex conjugate,

$$s^2 + 40s + 2898.5 \Rightarrow (s-\alpha)^2 + \beta^2$$

$$\Rightarrow (s+20)^2 + 50^2$$

Now,

$$I(s) = \frac{400}{(s+20)^2 + 50^2}$$

$$\frac{\omega}{(s+a)^2+\omega^2} = e^{-at}\sin\omega t$$

$\times$ and $\div$ by 50

$$I(s) = \frac{400}{[(s+20)^2 + 50^2]} \times \frac{50}{50}$$

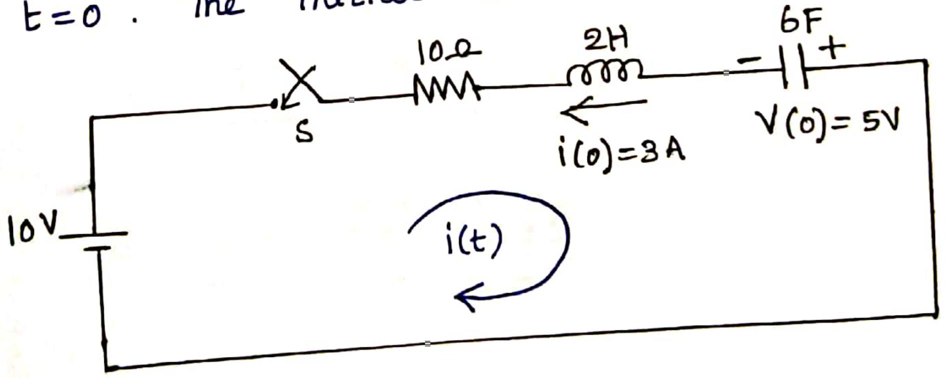
$\therefore$ Take Inverse laplace transform.

$$= \frac{400}{50} e^{-20t} \sin 50t$$

$$\boxed{i(t) = 8e^{-20t} \sin 50t}$$

---

4) In the circuit shown in Fig. Find the transient current when the switch is closed at $t=0$. The initial conditions are shown.



Solution:-

$$10\,i(t) + \frac{2di(t)}{dt} + \frac{1}{6}\int i(t).dt - V_0 - 10 = 0$$

$$10\,i(t) + 2\frac{di(t)}{dt} + \frac{1}{6}\int i(t).dt - 5 - 10 = 0$$

$$10\,i(t) + 2.\frac{di(t)}{dt} + \frac{1}{6}\int i(t).dt = 15$$

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Take Laplace transform,

$$10I(s) + 2[sI(s) - i(0)] + \frac{1}{6s} I(s) = \frac{15}{s}.$$

$$10I(s) + 2[sI(s) - (-3)] + \frac{1}{6s} I(s) = \frac{15}{s}.$$

$$10I(s) + 2sI(s) + 6 + \frac{1}{6s} I(s) = \frac{15}{s}.$$

$$I(s)\left[10 + 2s + \frac{1}{6s}\right] = \frac{15}{s} - 6$$

$$I(s)\left[10 + 2s + \frac{1}{6s}\right] = \frac{15 - 6s}{s}$$

$$I(s) = \frac{15 - 6s}{s\left(10 + 2s + \frac{1}{6s}\right)}$$

$$I(s) = \frac{15 - 6s}{10s + 2s^2 + \frac{1}{6}}$$

$$I(s) = \frac{15 - 6s}{2\left[\frac{10s}{2} + s^2 + \frac{1}{2\times6}\right]}$$

$$I(s) = \frac{\frac{15 - 6s}{2}}{s^2 + 5s + 0.0833}$$

$$I(s) = \frac{7.5 - 3s}{s^2 + 5s + 0.0833}$$

Dr,

$$s^2 + 5s + 0.0833$$

$$\boxed{s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$s = \frac{-5 \pm \sqrt{5^2 - 4\times 0.083}}{2}$$

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$$S = \frac{-5 \pm \sqrt{25 - 0.332}}{2}$$

$$S = \frac{-5 \pm 4.966}{2}$$

$$S = \frac{-5 + 4.966}{2}$$

$$S = -0.017$$

$$S = \frac{-5 - 4.966}{2}$$

$$S = -4.983$$

$$s^2 + 5s + 0.0833 \Rightarrow [s - (-0.017)][s - (-4.983)]$$

Now, using Partial fraction,

$$I(S) = \frac{7.5 - 3S}{S^2 + 5S + 0.0833} = \frac{A}{(S + 0.017)} + \frac{B}{(S + 4.983)}$$

$$\Rightarrow \frac{7.5 - 3S}{\cancel{s^2 + 5s + 0.0833}} = \frac{A(S + 4.983) + B(S + 0.017)}{\cancel{(S + 0.017)(S + 4.983)}}$$

$$\Rightarrow 7.5 - 3S = A(S + 4.983) + B(S + 0.017)$$

Put $s = -4.983$

$$7.5 - 3(-4.983) = B(-4.983 + 0.017)$$

$$22.449 = B(-4.996)$$

$$B = \frac{22.449}{-4.996}$$

$$\boxed{B = -4.52}$$

Put $S = -0.017$

$$7.5 - 3(-0.017) = A(-0.017 + 4.983)$$

$$7.551 = A(4.966)$$

$$A = \frac{7.551}{4.966}$$

$$\boxed{A = 1.52}$$

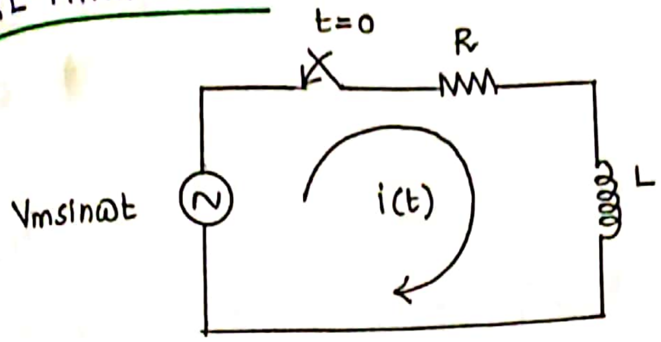
Now,
$$I(S) = \frac{1.52}{S + 0.017} - \frac{4.52}{S + 4.983}$$

Take Inverse Laplace transform

$$\boxed{i(t) = 1.52e^{-0.017t} - 4.52e^{-4.983t}}$$

TRASIENT RESPONSE FOR AC CIRCUITS.

RL TRANSIENTS



Apply KVL

$$R\, i(t) + L.\frac{di(t)}{dt} = V_m \sin\omega t$$

Take Laplace transform,

$$R I(s) + L.s I(s) = V_m . \frac{\omega}{s^2+\omega^2}$$

$$I(s)\left[R + Ls\right] = \frac{V_m \omega}{s^2+\omega^2}$$

$$I(s) = \frac{V_m \omega}{(s^2+\omega^2)(R+Ls)}$$

$$I(s) = \frac{V_m \omega}{L\left(s+\frac{R}{L}\right)(s^2+\omega^2)}$$

$$I(s) = \frac{V_m \omega}{L\left(s+\frac{R}{L}\right)(s+j\omega)(s-j\omega)}$$

$$I(s) = \frac{\frac{V_m \omega}{L}}{\left(s+\frac{R}{L}\right)(s+j\omega)(s-j\omega)}$$

using
Partial fraction,

$$\frac{V_m\omega/L}{\left(s+\frac{R}{L}\right)(s+j\omega)(s-j\omega)} = \frac{A}{\left(s+\frac{R}{L}\right)} + \frac{B}{(s+j\omega)} + \frac{C}{(s-j\omega)}$$

$$\frac{V_m\omega/L}{\cancel{\left(s+\frac{R}{L}\right)(s+j\omega)(s-j\omega)}} = \frac{A(s+j\omega)(s-j\omega) + B\left(s+\frac{R}{L}\right)(s-j\omega) + C\left(s+\frac{R}{L}\right)(s+j\omega)}{\cancel{\left(s+\frac{R}{L}\right)(s+j\omega)(s-j\omega)}}$$

$$\frac{V_m\omega}{L} = A(s+j\omega)(s-j\omega) + B\left(s+\frac{R}{L}\right)(s-j\omega) + C\left(s+\frac{R}{L}\right)(s+j\omega) \quad \text{——(1)}$$

When $s=j\omega$

(1) ⇒

$$\frac{V_m\omega}{L} = C\left[j\omega+\frac{R}{L}\right]\left[j\omega+j\omega\right]$$

$$\frac{V_m\cancel{\omega}}{\cancel{L}} = C\left[\frac{j\omega L+R}{\cancel{L}}\right]\left[2j\cancel{\omega}\right]$$

$$V_m = C\left[2j^2\omega L + 2jR\right]$$

$$V_m = C\left[-2\omega L + 2jR\right]$$

$$C = \frac{V_m}{-2\omega L+2jR}$$

$$C = \frac{V_m}{-2(\omega L-jR)}$$

$$C = \frac{-V_m}{2}\cdot\frac{1}{(\omega L-jR)} \times \frac{(\omega L+jR)}{(\omega L+jR)}$$

$$C = \frac{-V_m(\omega L+jR)}{2\left[(\omega L)^2+R^2\right]}$$

$$\boxed{C = \frac{-V_m(\omega L+jR)}{2\left[R^2+\omega^2L^2\right]}}$$

When $s=-j\omega$

(1) ⇒

$$\frac{V_m\omega}{L} = B\left(-j\omega+\frac{R}{L}\right)(-j\omega-j\omega)$$

$$\frac{V_m\cancel{\omega}}{\cancel{L}} = B\left(\frac{-j\omega L+R}{\cancel{L}}\right)(-2j\cancel{\omega})$$

$$V_m = B\left(2j^2\omega L - 2jR\right)$$

$$V_m = B\left(2(-1)\omega L - 2jR\right)$$

$$V_m = -2B\left[\omega L+jR\right]$$

$$B = \frac{V_m}{-2} \cdot \frac{1}{(\omega L + jR)}$$

$$B = \frac{-V_m}{2} \cdot \frac{1}{(\omega L + jR)} \times \frac{(\omega L - jR)}{(\omega L - jR)}$$

$$\boxed{B = \frac{-V_m(\omega L - jR)}{2[\omega^2 L^2 + R^2]}}$$

when $s = \frac{-R}{L}$

(1) $\Rightarrow$ $$\frac{V_m \omega}{L} = A\left(-\frac{R}{L} + j\omega\right)\left(-\frac{R}{L} - j\omega\right)$$

$$\frac{V_m \omega}{\cancel{L}} = A\left(\frac{-R + j\omega L}{\cancel{L}}\right)\left(\frac{-R - j\omega L}{L}\right)$$

$$V_m \omega = A\left[\frac{R^2 + \omega^2 L^2}{L}\right]$$

$$\boxed{A = \frac{V_m \omega L}{R^2 + \omega^2 L^2}}$$

Now,

$$I(s) = \frac{\frac{V_m \omega L}{R^2 + \omega^2 L^2}}{\left(s + \frac{R}{L}\right)} - \frac{\frac{V_m(\omega L - jR)}{2[R^2 + \omega^2 L^2]}}{(s + j\omega)} - \frac{\frac{V_m(\omega L + jR)}{2[R^2 + \omega^2 L^2]}}{(s - j\omega)}$$

$$I(s) = \frac{V_m \omega L}{R^2 + \omega^2 L^2} \cdot \frac{1}{\left(s + \frac{R}{L}\right)} - \frac{V_m}{2[R^2 + \omega^2 L^2]}\left[\frac{\omega L - jR}{s + j\omega} + \frac{\omega L + jR}{s - j\omega}\right]$$

Take Inverse Laplace Transform.

$$i(t) = \frac{V_m \omega L}{R^2 + \omega^2 L^2} \cdot e^{-Rt/L} - \frac{V_m}{2[R^2 + \omega^2 L^2]}\left[(\omega L - jR)e^{-j\omega t} + (\omega L + jR)e^{j\omega t}\right]$$

$$i(t) = \frac{V_m \omega L}{R^2 + \omega^2 L^2} e^{-Rt/L} - \frac{V_m}{2[R^2 + \omega^2 L^2]}\left[(\omega L - jR)(\cos\omega t - j\sin\omega t) + (\omega L + jR)(\cos\omega t + j\sin\omega t)\right]$$

$$i(t) = \frac{V_m \omega L}{R^2+\omega^2 L^2} e \quad - \frac{V_m}{2[R^2+\omega^2L^2]} \begin{bmatrix} \omega L \cos\omega t - \cancel{j\omega L \sin\omega t} - \cancel{jR\cos\omega t} \\ - R\sin\omega t + \omega L\cos\omega t + \\ \cancel{+j\omega L \sin\omega t} + \cancel{jR\cos\omega t} - R\sin\omega t \end{bmatrix}$$

$$i(t) = \frac{V_m \omega L}{R^2+\omega^2 L^2} e^{-Rt/L} - \frac{V_m}{2[R^2+\omega^2L^2]} \left[2\omega L\cos\omega t - 2R\sin\omega t\right]$$

$$i(t) = \frac{V_m \omega L}{R^2+\omega^2 L^2} e^{-Rt/L} - \frac{V_m \cancel{(2)}}{\cancel{2}[R^2+\omega^2L^2]} \left[\omega L\cos\omega t - R\sin\omega t\right]$$

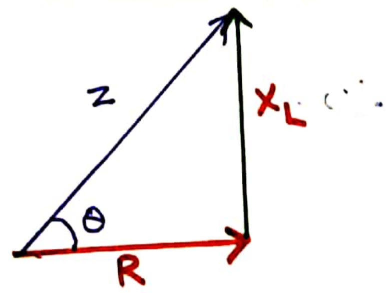
$$i(t) = \frac{V_m \omega L}{R^2+\omega^2 L^2} \cdot e^{-Rt/L} - \frac{V_m \omega L\cos\omega t}{R^2+\omega^2L^2} + \frac{V_m R\sin\omega t}{R^2+\omega^2L^2}$$

$$i(t) = \frac{V_m}{R^2+\omega^2L^2}\left[\omega L e^{-Rt/L} - \omega L\cos\omega t + R\sin\omega t\right]$$

$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}\cdot\sqrt{R^2+\omega^2L^2}}\left[\omega L e^{-Rt/L} - \omega L\cos\omega t + R\sin\omega t\right]$$

$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left[\frac{\omega L e^{-Rt/L}}{\sqrt{R^2+\omega^2L^2}} - \frac{\omega L\cos\omega t}{\sqrt{R^2+\omega^2L^2}} + \frac{R\sin\omega t}{\sqrt{R^2+\omega^2L^2}}\right]$$

Impedance Triangle.



$X_L = \omega L$

$Z = \sqrt{R^2+X_L^2}$

$Z = \sqrt{R^2+\omega^2L^2}$

$$i(t) = \frac{V_m}{Z}\left[\frac{\omega L e^{-Rt/L}}{Z} - \frac{\omega L\cos\omega t}{Z} + \frac{R\sin\omega t}{Z}\right]$$

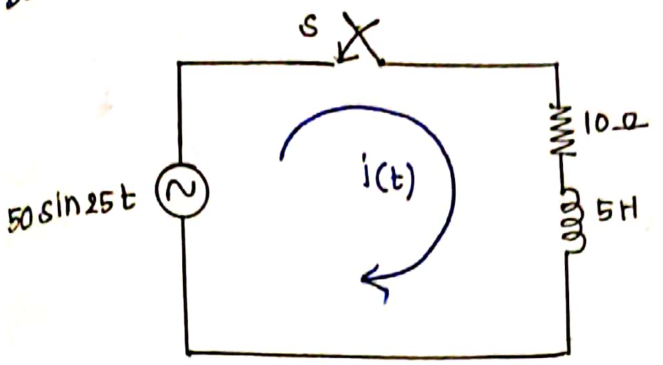
From Fig.

$$\cos\theta = \frac{R}{Z}$$

$$\sin\theta = \frac{X_L}{Z} \Rightarrow \frac{\omega L}{Z}$$

$$i(t) = \frac{V_m}{Z}\left[e^{-Rt/L}\cdot\sin\theta - \sin\theta\cos\omega t + \cos\theta\cdot\sin\omega t\right]$$

1) The circuit shown in Figure consists of series RL elements. The sine wave is applied to the circuit when the switch is closed at $t=0$. Determine the current $i(t)$.



Apply KVL

$$10\,i(t) + 5.\frac{di(t)}{dt} = 50 \sin 25t$$

Take Laplace transform.

$$10 I(s) + 5.\, s I(s) = 50 \cdot \frac{25}{s^2+25^2}$$

$\div 5$

$$2 I(s) + s I(s) = 10 \cdot \frac{25}{s^2+25^2}$$

$$I(s)\left[2+s\right] = \frac{250}{s^2+25^2}$$

$$I(s) = \frac{250}{(s^2+25^2)(s+2)}$$

$$I(s) = \frac{250}{(s+j25)(s-j25)(s+2)}$$

using partial fraction

$$\frac{250}{(s+j25)(s-j25)(s+2)} = \frac{A}{(s+j25)} + \frac{B}{(s-j25)} + \frac{C}{(s+2)}$$

$$\frac{250}{\cancel{(s+j25)(s-j25)}(s+2)} = \frac{A(s-j25)(s+2) + B(s+j25)(s+2) + C(s+j25)(s-j25)}{\cancel{(s+j25)(s-j25)}(s+2)}$$

$$250 = A(s-j25)(s+2) + B(s+j25)(s+2) + C(s+j25)(s-j25)$$

Put $s=-2$

$$250 = C\,(-2+j25)(-2-j25)$$

$$250 = C\left[(-2)^2 + 25^2\right]$$

$$250 = C(629)$$

$$C = \frac{250}{629}$$

$$\boxed{C = 0.397}$$

Put $s = -j25$

$$250 = A(-j25 - j25)(-j25+2)$$

$$250 = A(-j50)(2-j25)$$

$$250 = A\left[-1250 - 100i\right]$$

$$A = \frac{250}{-1250 - 100j}$$

$$\boxed{A = -0.198 + 0.015j}$$

Put $s = j25$

$$250 = B(j25 + j25)(j25 + 2)$$

$$250 = B(j50)(2+j25)$$

$$250 = B\left[-1250 + 100i\right]$$

$$B = \frac{250}{-1250 + 100i}$$

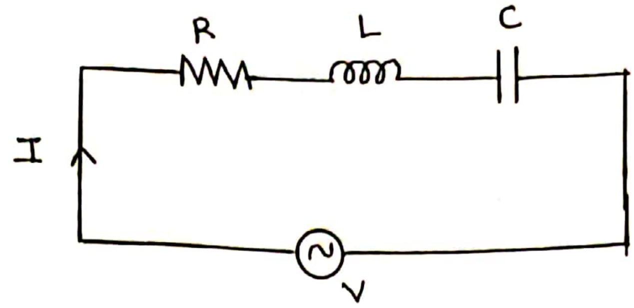
$$\boxed{B = -0.198 - 0.015j}$$

$$I(s) = \frac{-0.198 + j0.015}{s + j25} - \frac{(0.198 + 0.015j)}{s - j25} + \frac{0.397}{s+2}$$

Take Inverse Laplace transform.

$$\boxed{i(t) = \left[-0.198 + j0.015\right]e^{-25t} - \left[0.198 + j0.015\right]e^{25t} + 0.397e^{-2t}}$$

RESONANCE IN SERIES AC CIRCUITS



Apply KVL

$$V = IR + jIX_L - jIX_C$$

$$V = I(R + jX_L - jX_C)$$

$$\frac{V}{I} = R + j(X_L - X_C)$$

Impedance, $\boxed{Z = R + j(X_L - X_C)}$

Here, When

$X_L > X_C \Rightarrow$ The circuit is Inductive $\Rightarrow$ Total current lag applied voltage

$X_C > X_L \Rightarrow$ The circuit is capacitive $\Rightarrow$ Total current lead applied voltage

W.K.T.

$$X_L = 2\pi f L \qquad X_C = \frac{1}{2\pi f C}$$

$$X_L = \omega L \qquad X_C = \frac{1}{\omega C}$$

At certain frequency $\boxed{X_L = X_C}$

✓ When $X_L = X_C$. The circuit is said to be in resonance.

✓ At resonance condition Impedance (Z) is minimum and is equal to resistance ie) $Z = R$

∴ At resonance, current is maximum

$$I = \frac{V}{R}$$

RESONANT FREQUENCY (f_r)

The frequency at which resonance occurs is called resonant frequency.

When

$$X_L = X_C$$

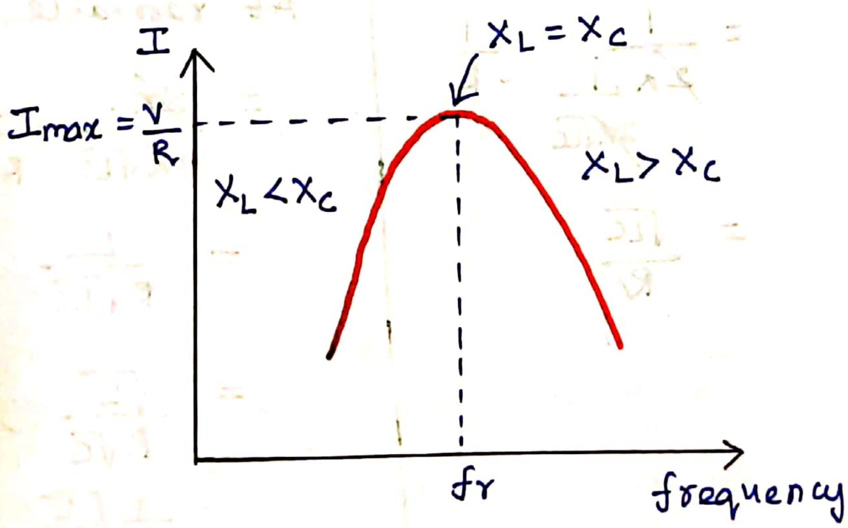
$$2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{(2\pi C)(2\pi L)}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$



By Applying ohms law,
the voltage across each element in the circuit are

$V_R = IR \angle 0^\circ$ — leading I by 90°

$V_L = IX_L \angle 90^\circ$

$V_C = IX_C \angle -90^\circ$ — lagging I by 90°

Voltage across L & C must have same magnitude and 180° out of phase.

QUALITY FACTOR (Q)

↙ Diagram Q. F

→ In series resonating circuit, Q factor is defined as the ratio of the voltage across the inductor or capacitor to the applied voltage.

$$\boxed{Q = \frac{V_L}{V} = \frac{V_C}{V}}$$

→ It is also the voltage magnification in the circuit at resonance. ie) At resonance, the voltage across the inductor and capacitor are magnified by Q times.

→ It is said to be high-Q circuit when its quality factor is equal to or greater than 10. $(Q \geq 10)$

$$Q = \frac{V_L}{V}$$

$$= \frac{IX_L}{IR}$$

$$\boxed{Q = \frac{X_L}{R}}$$ (at resonance)

$$Q = \frac{V_C}{V}$$

$$= \frac{IX_C}{IR}$$

$$\boxed{Q = \frac{X_C}{R}}$$ (at resonance)

→ $$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$\boxed{\text{Quality factor} = \frac{f_r}{f_H - f_L}}$$